COBEM 2025

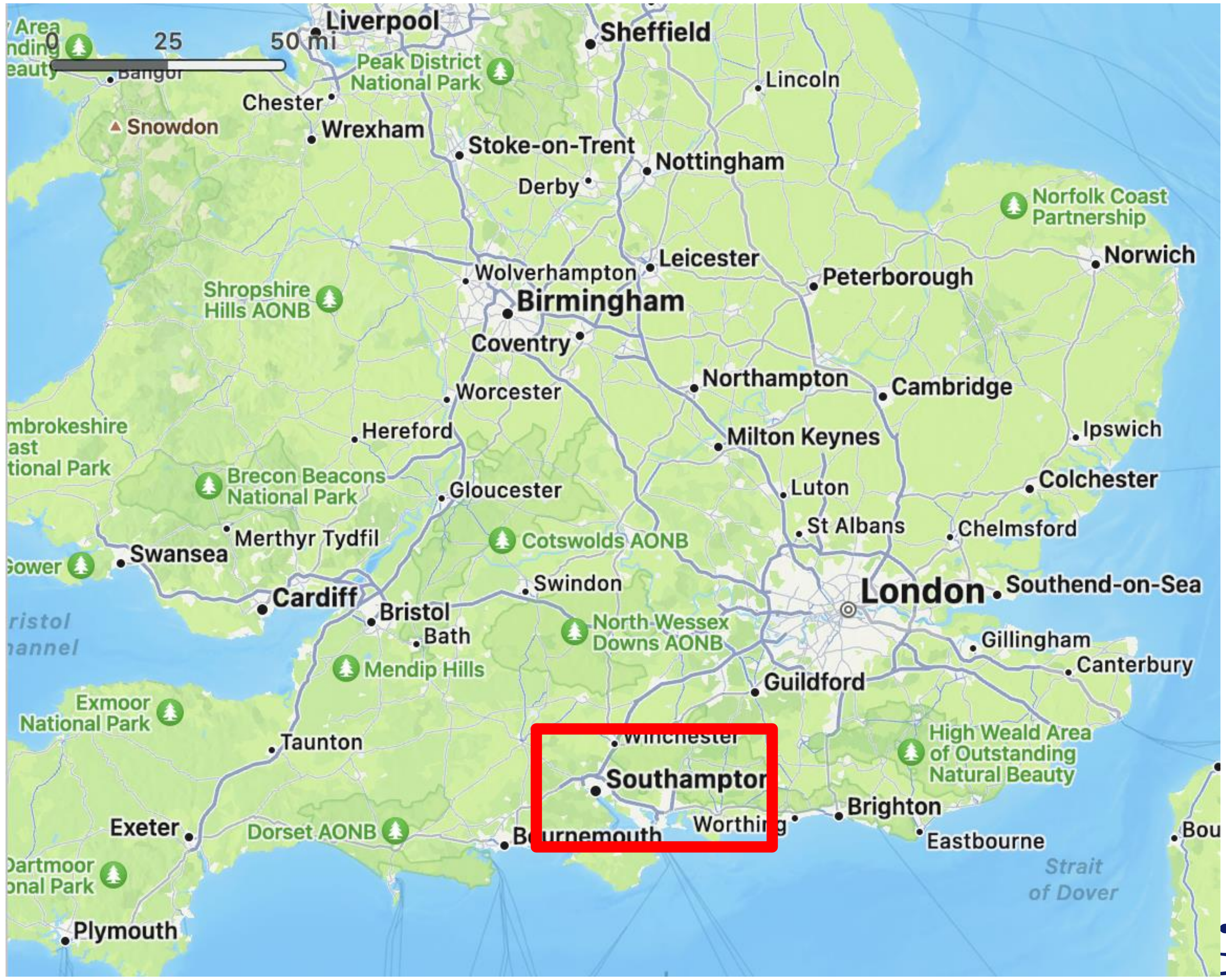
28th International Congress of Mechanical Engineering

Unravelling the Complexity of Vibroimpact Systems

Daniil Yurchenko

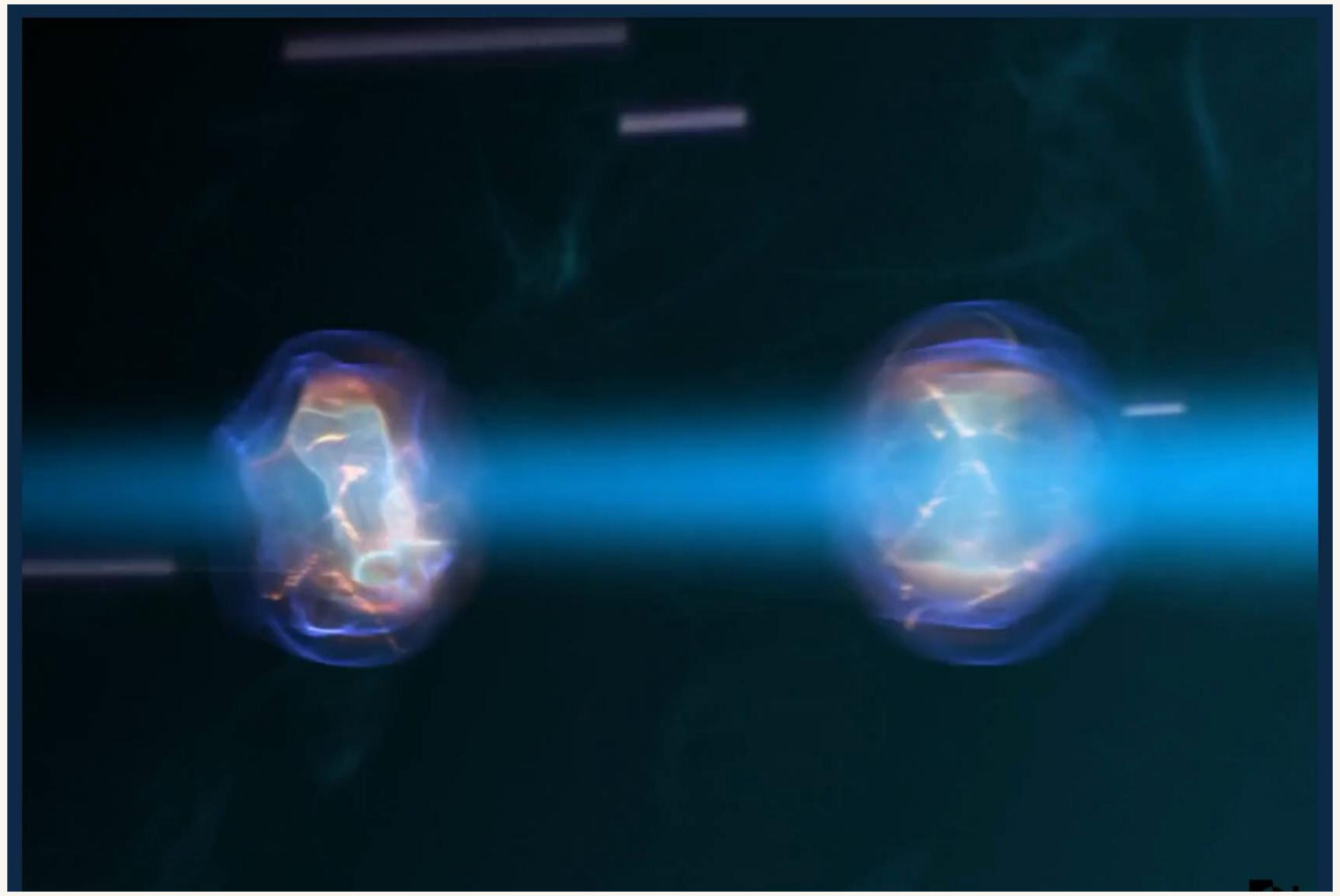
Institute of Sound and Vibration Research, University of Southampton, UK


NORWAY
Bergen
Oslo
Stockholm
Baltic Sea
North Sea
DENMARK
Copenhagen
LITHUA
UNITED KINGDOM
IRELAND
Birmingham
London
NETHERLANDS
Hamburg
Berlin
POLAND
GERMANY
Wrocław
Katowice
BELGIUM
LUXEMBOURG
CZECHIA
SLOVAKIA
Paris
Vienna
Munich
AUSTRIA
HUNGARY
ATLANTIC BASIN
Bay of Biscay
FRANCE
Lyon
ALPS
SLOVENIA
CROATIA
Milan
Turin
Bordeaux
BOSNIA AND HERZEGOVINA
Toulouse
Marseille
ANDORRA
Rome
SPAIN
Madrid
Barcelona
Naples
PORTUGAL
Sardinia
Sicily
Tunis
Algiers
Tangier
MALTA


0 25 50 mi
Liverpool
Sheffield
Peak District National Park
Lincoln
Chester
Snowdon
Wrexham
Stoke-on-Trent
Nottingham
Derby
Norfolk Coast Partnership
Norwich
Leicester
Wolverhampton
Peterborough
Shropshire Hills AONB
Birmingham
Coventry
Northampton
Cambridge
Worcester
Hereford
Milton Keynes
Ipswich
Brecon Beacons National Park
Gloucester
Luton
Colchester
St Albans
Chelmsford
Merthyr Tydfil
Cotswolds AONB
Swansea
Swindon
London
Southend-on-Sea
Cardiff
Bristol
North Wessex Downs AONB
Bath
Gillingham
Canterbury
Mendip Hills
Guildford
Exmoor National Park
Taunton
High Weald Area of Outstanding Natural Beauty
Southampton
Brighton
Exeter
Dorset AONB
Worthing
Eastbourne
Strait of Dover
Plymouth

Content

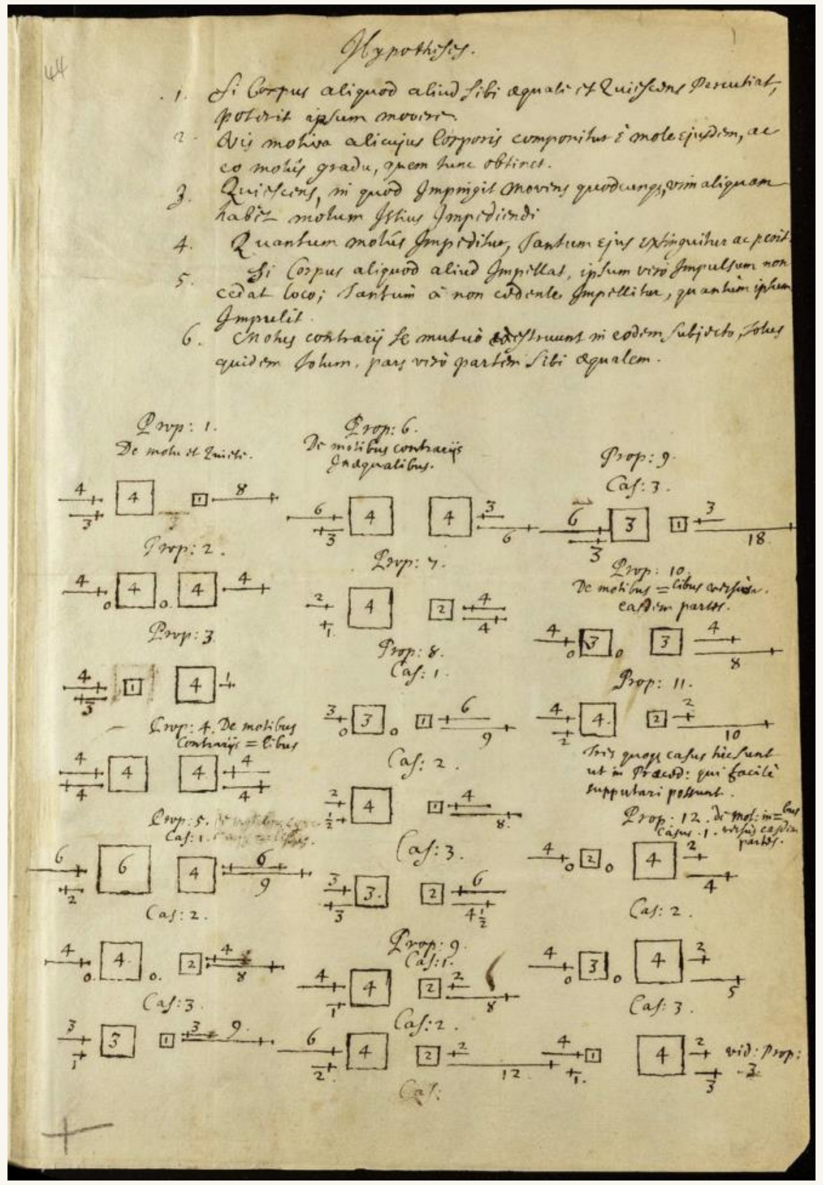
- Historic overview
- Impact dynamics
- Vibro-impact capsule
- Computer-assisted method
- Conclusion



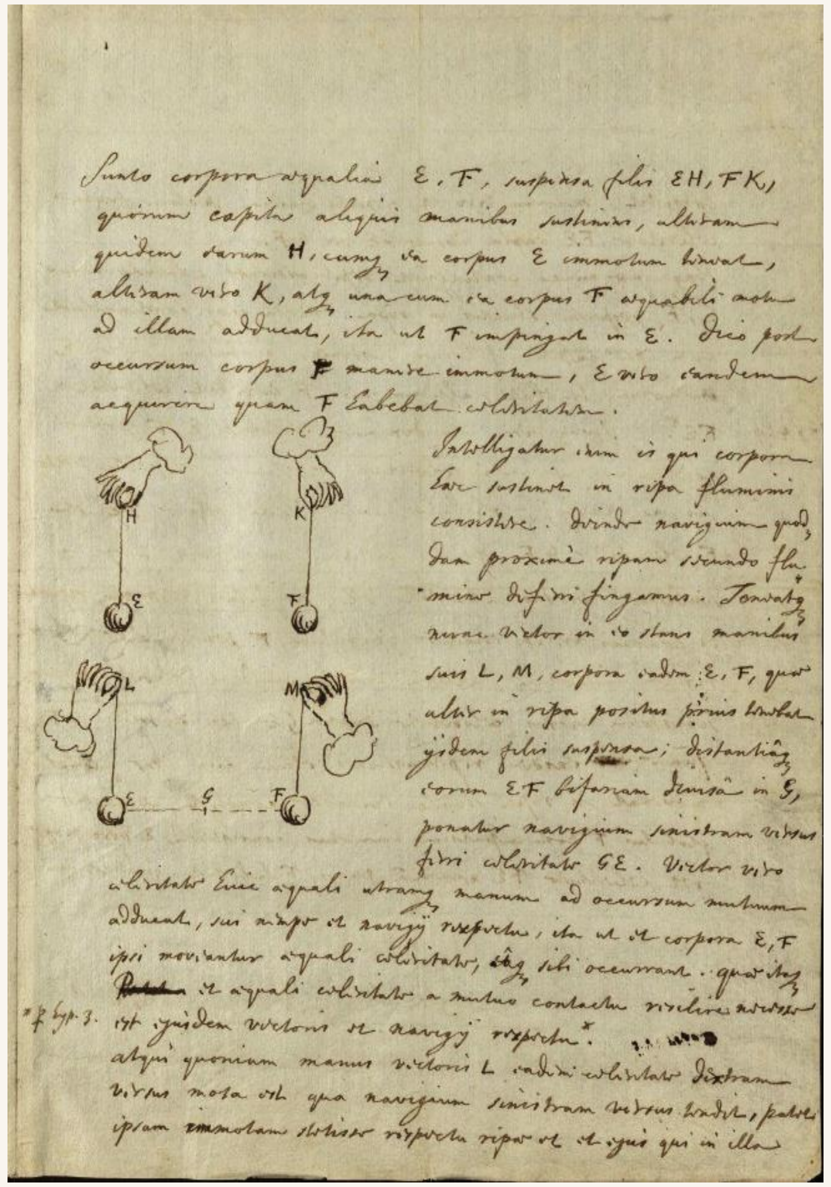
Historic overview

COBEM 2025
28th International Congress of Mechanical Engineering

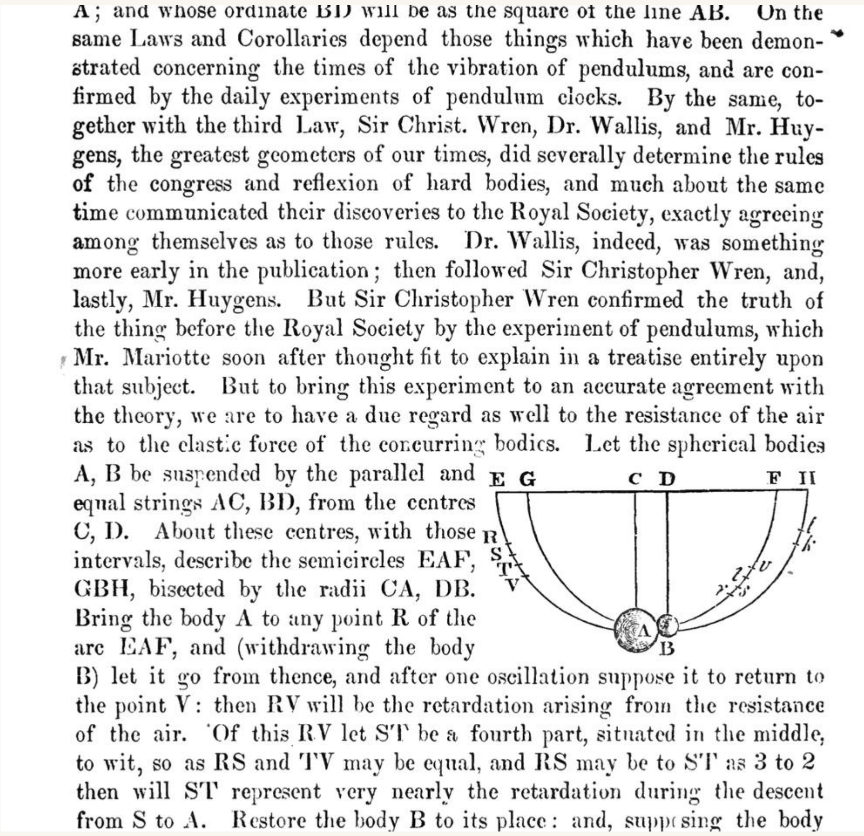
The law of nature concerning the collision of bodies
C. Wren, 1668



Paper on motion
W. Croone, 1668



On the motion of bodies resulting from impact
C. Huygens, 1668

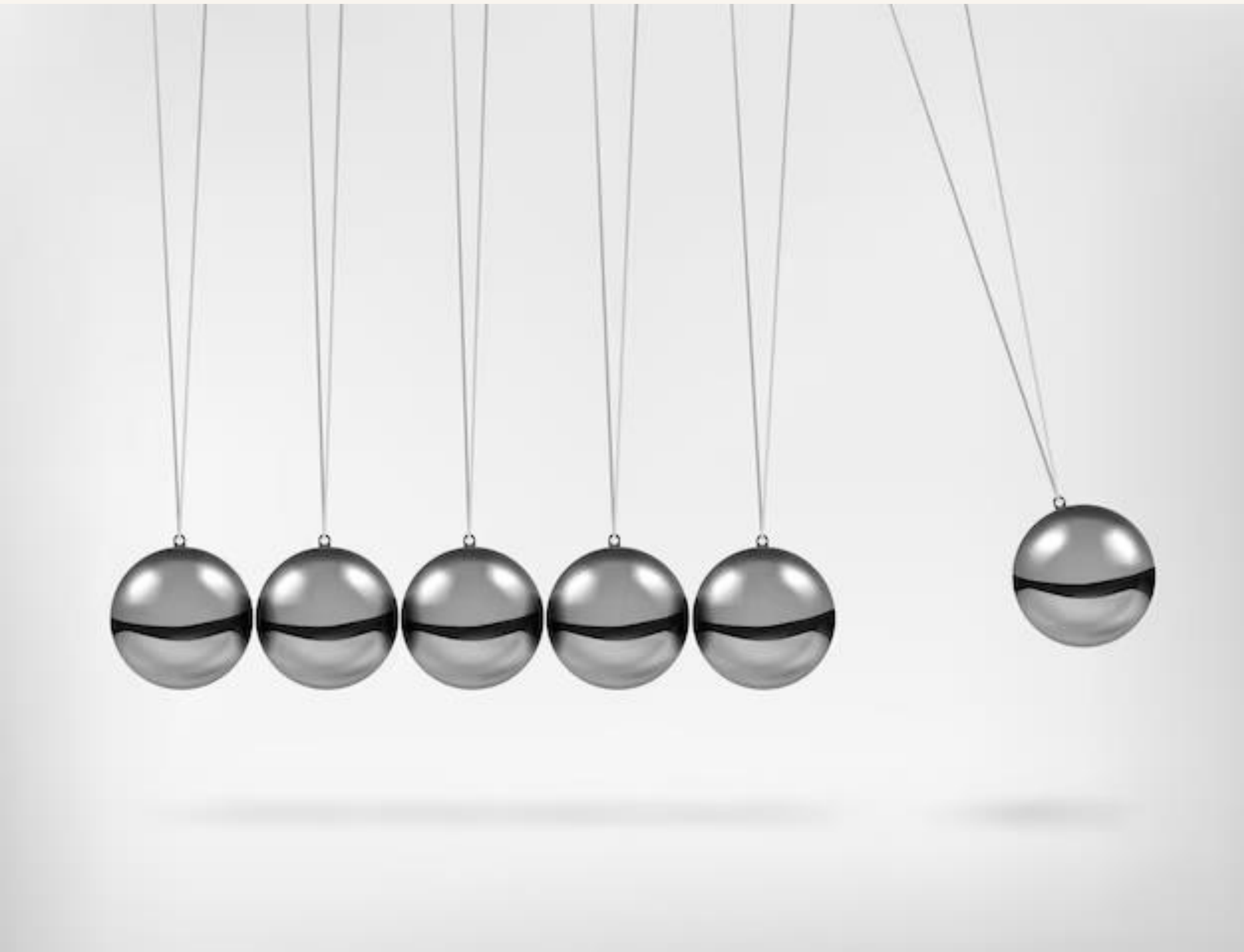

A; and whose ordinate BD will be as the square of the line AB. On the same Laws and Corollaries depend those things which have been demonstrated concerning the times of the vibration of pendulums, and are confirmed by the daily experiments of pendulum clocks. By the same, together with the third Law, Sir Christ. Wren, Dr. Wallis, and Mr. Huygens, the greatest geometers of our times, did severally determine the rules of the congress and reflexion of hard bodies, and much about the same time communicated their discoveries to the Royal Society, exactly agreeing among themselves as to those rules. Dr. Wallis, indeed, was something more early in the publication; then followed Sir Christopher Wren, and, lastly, Mr. Huygens. But Sir Christopher Wren confirmed the truth of the thing before the Royal Society by the experiment of pendulums, which Mr. Mariotte soon after thought fit to explain in a treatise entirely upon that subject. But to bring this experiment to an accurate agreement with the theory, we are to have a due regard as well to the resistance of the air as to the elastic force of the concurring bodies. Let the spherical bodies A, B be suspended by the parallel and equal strings AC, BD, from the centres C, D. About these centres, with those intervals, describe the semicircles EAF, GBH, bisected by the radii CA, DB. Bring the body A to any point R of the arc EAF, and (withdrawing the body B) let it go from thence, and after one oscillation suppose it to return to the point V: then RV will be the retardation arising from the resistance of the air. Of this RV let ST be a fourth part, situated in the middle, to wit, so as RS and TV may be equal, and RS may be to ST as 3 to 2 then will ST represent very nearly the retardation during the descent from S to A. Restore the body B to its place: and, supposing the body

The Principia (I. Newton, 1686)
Translated by A Motte
1846

COBEM 2025

Historic overview

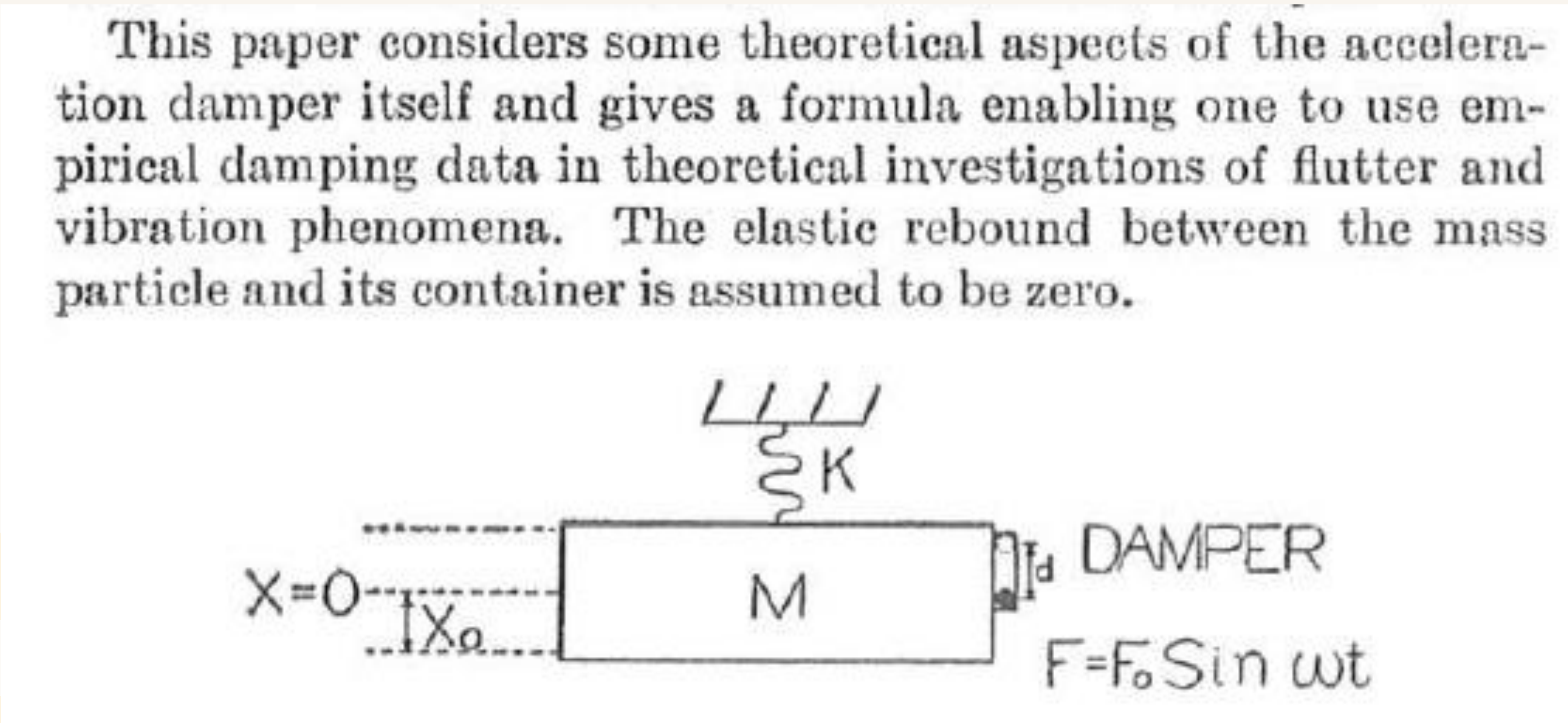
Newton cradle



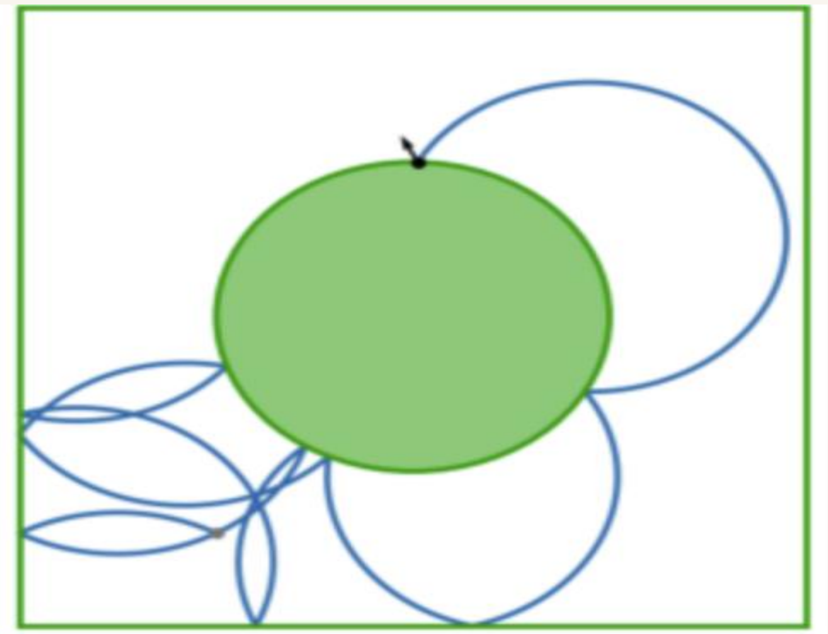
Invented by Edme Mariotte

Historic overview

P. Lieber and D.P. Jensen; 1945


This paper considers some theoretical aspects of the acceleration damper itself and gives a formula enabling one to use empirical damping data in theoretical investigations of flutter and vibration phenomena. The elastic rebound between the mass particle and its container is assumed to be zero.

Sinai billiard; 1963



Vibro-impact systems have rich dynamical behavior

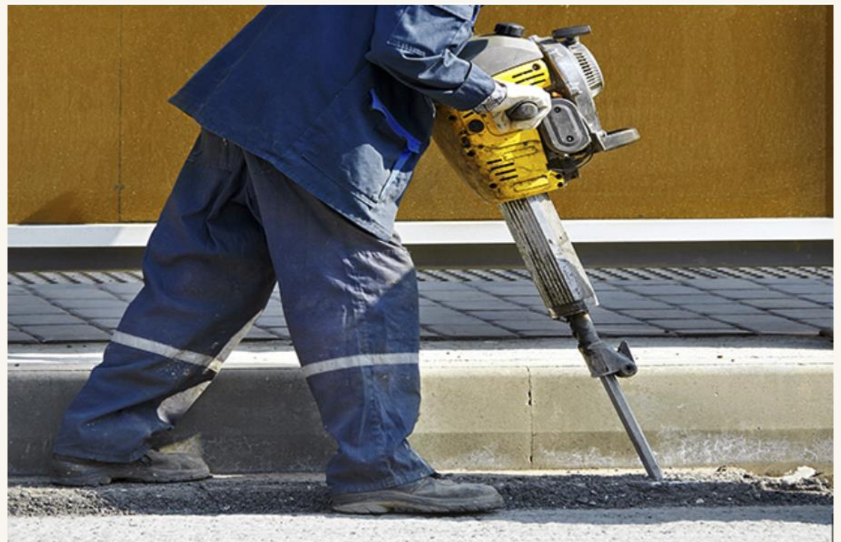
- Chaotic motion
- Possesses classical and non-classical bifurcations, including Symmetry breaking, Grazing, Border-Collision and Chattering bifurcations
- Use special methods developed for VI systems (Zhuravlev's transformation, Mapping approach)
- Motivated targeted energy transfer and nonlinear energy sinks

Impacting systems

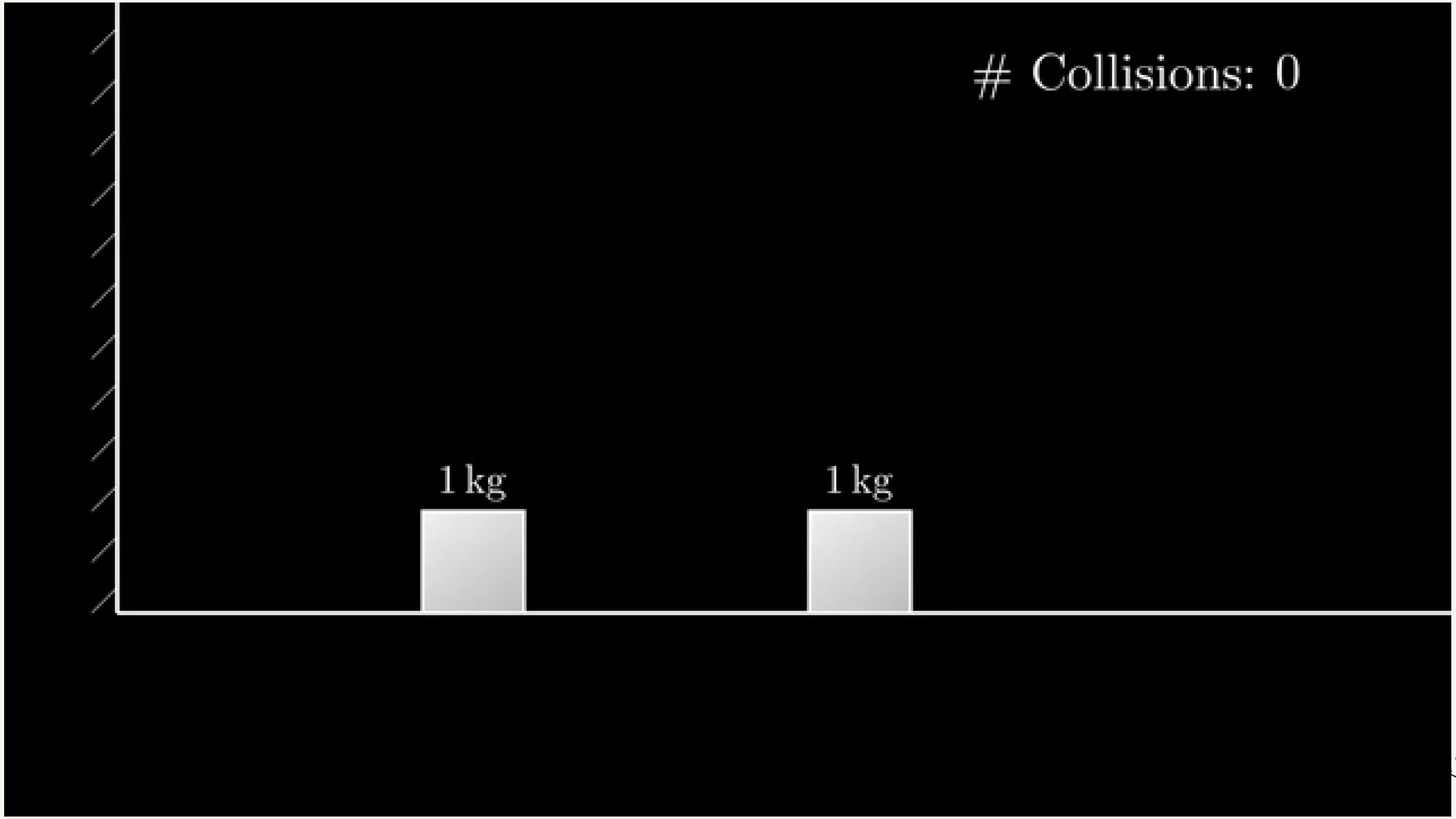
Basketball, Baseball, Boules, Bowlin, Cricket, Croquet, Billiard, Football, Golf, Bandy Field Hockey, Netball, Polo, Table football, Table Tennis, Tennis, Volleyball and many others

Impacting systems



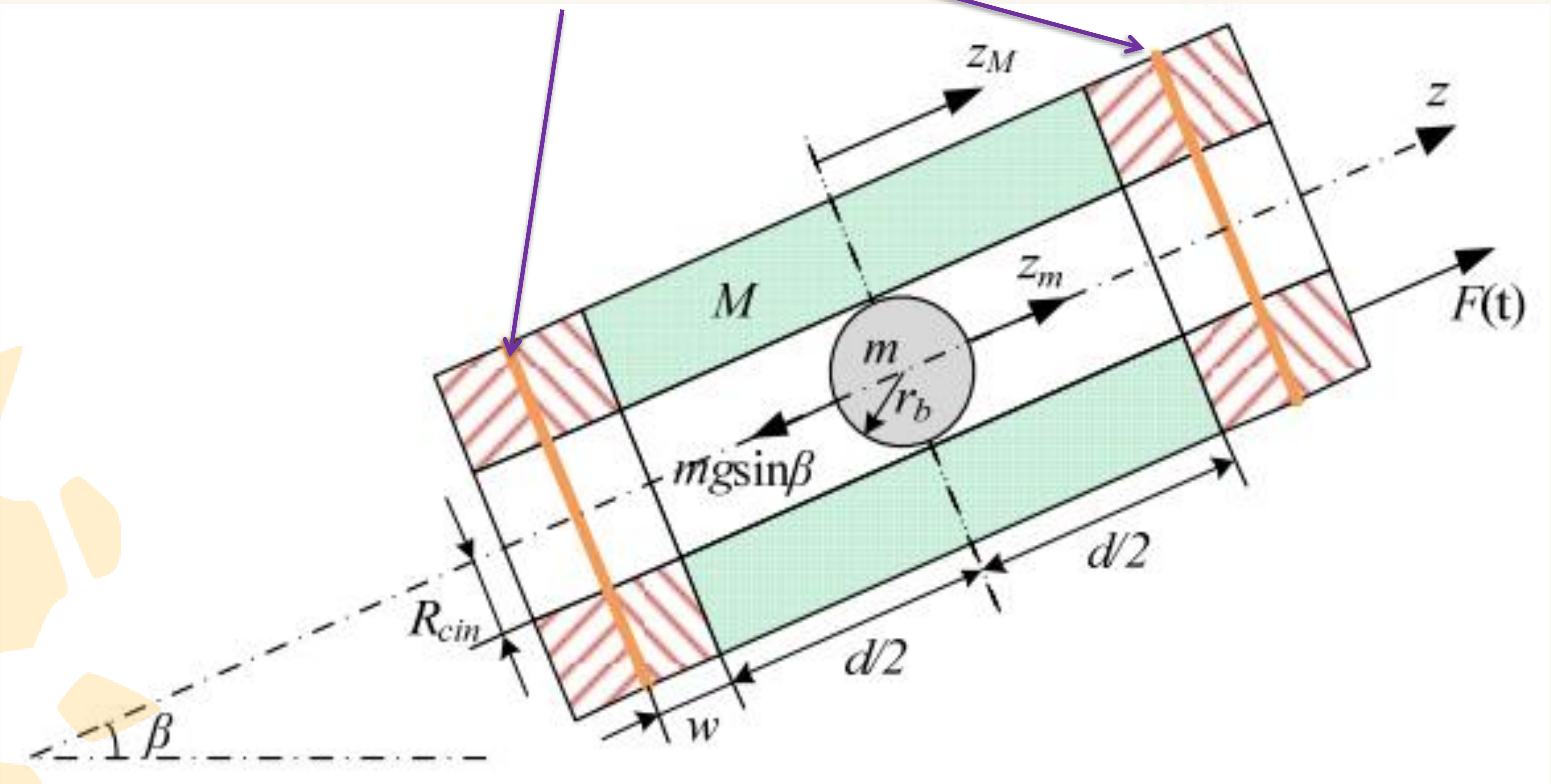


Impacting systems



VI-CAPSULE

DE membranes



No damping and no friction are assumed

Motion of the tube is:

$$z''_M(t) = \frac{A\cos(2\pi f_0 t)}{M}$$

Motion of the ball is:

$$z''_m(t) = -g\sin\beta$$

$$d = \frac{sM\omega^2}{A\pi^2}$$

The impact conditions are:

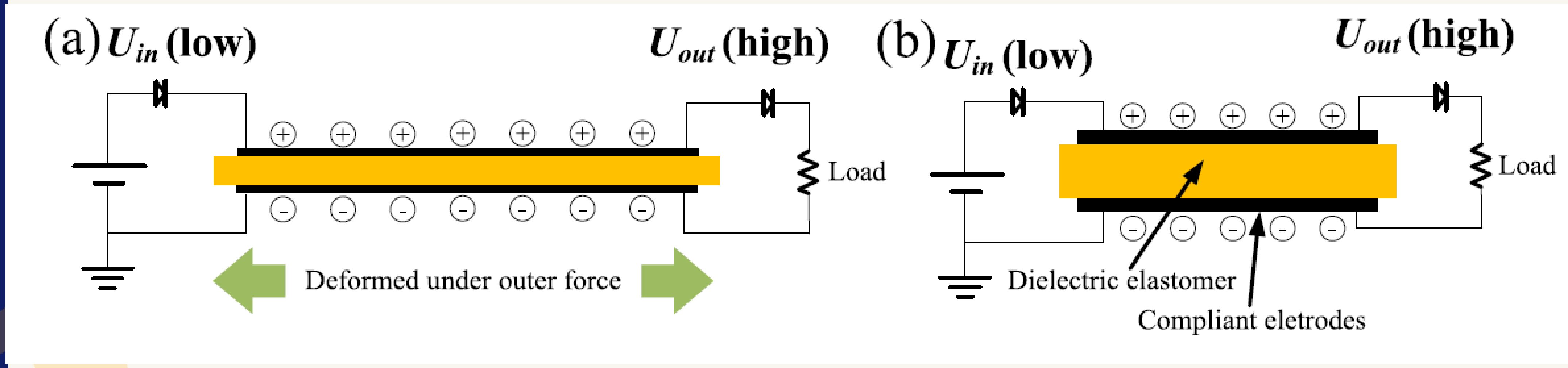
$\Delta z = z_M - z_m = s/2$ for impacts on the left

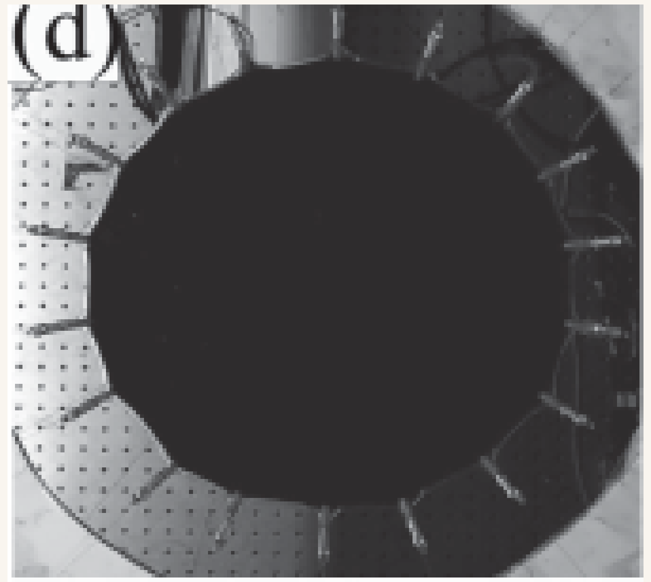
$\Delta z = z_M - z_m = -s/2$ for impacts on the right

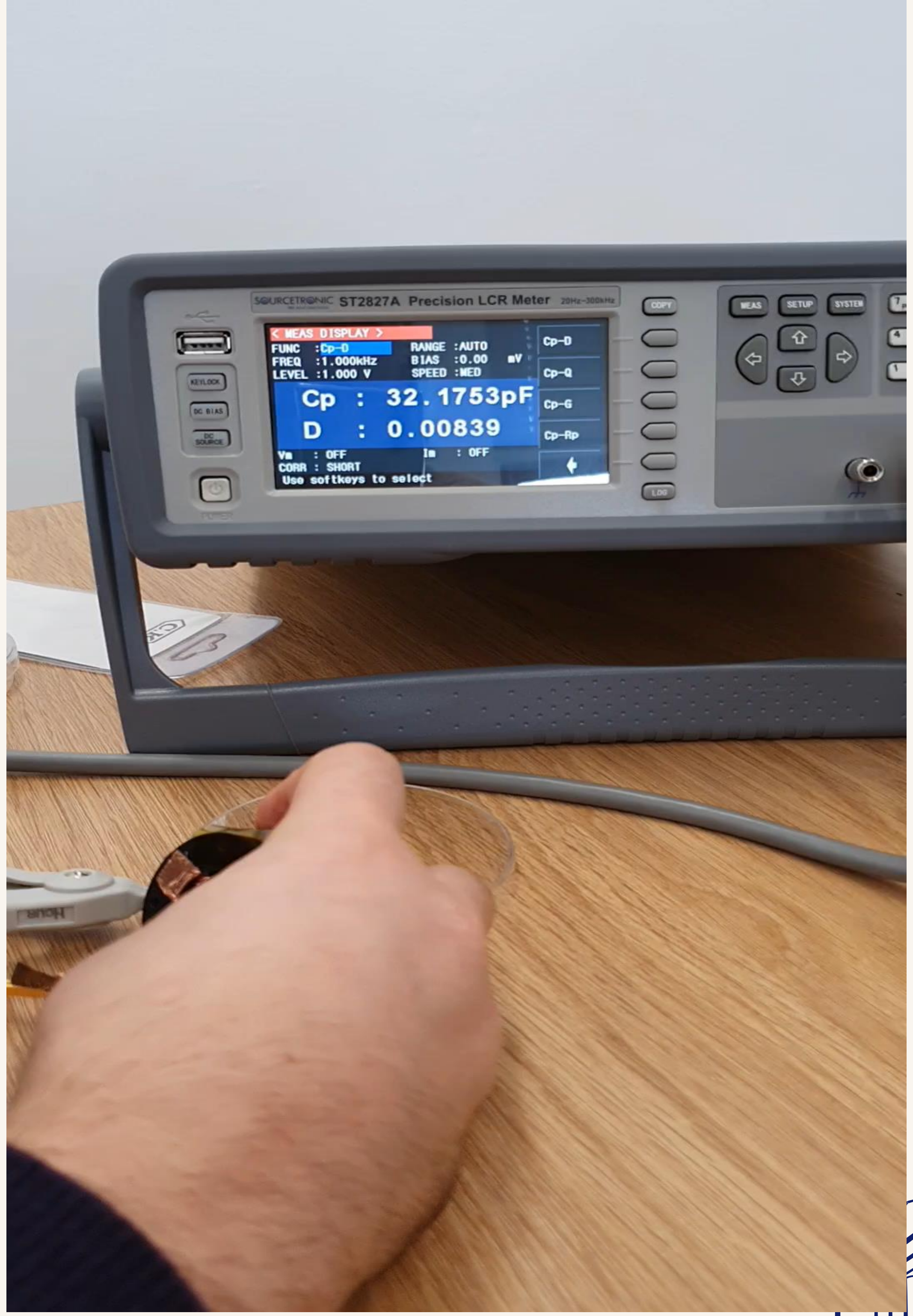
COBEM 2025

VI-CAPSULE

$$Q = CV, C = \frac{\varepsilon\varepsilon_0 Vol}{z^2}, \mathrm{d}Q = VdC$$







COBEM 2025

Maps approach

$$P_1 : \partial B \mapsto \partial T, \quad P_2 : \partial T \mapsto \partial B,$$
$$P_3 : \partial B \mapsto \partial B, \quad P_4 : \partial T \mapsto \partial T.$$

We consider 1:1 motion

P_1P_2

We consider 2:1 motion

$P_1P_2P_3$

To understand the conditions that lead to the higher efficiency and using the design parameters to sustain this motion

Attracting (stable) behavior for r<1

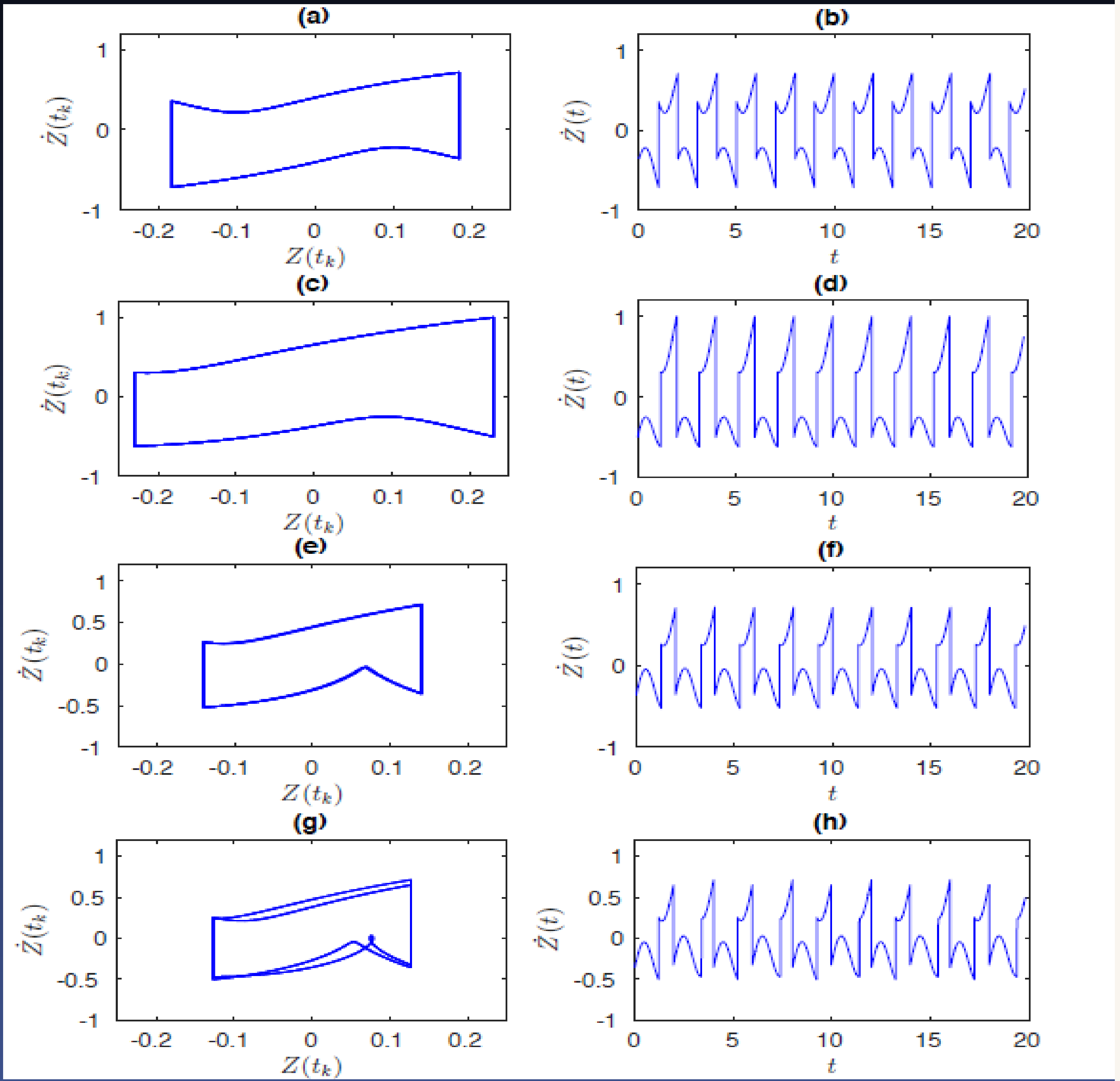
Symmetric

$\beta = 0, \|\hat{F}\| = 3$

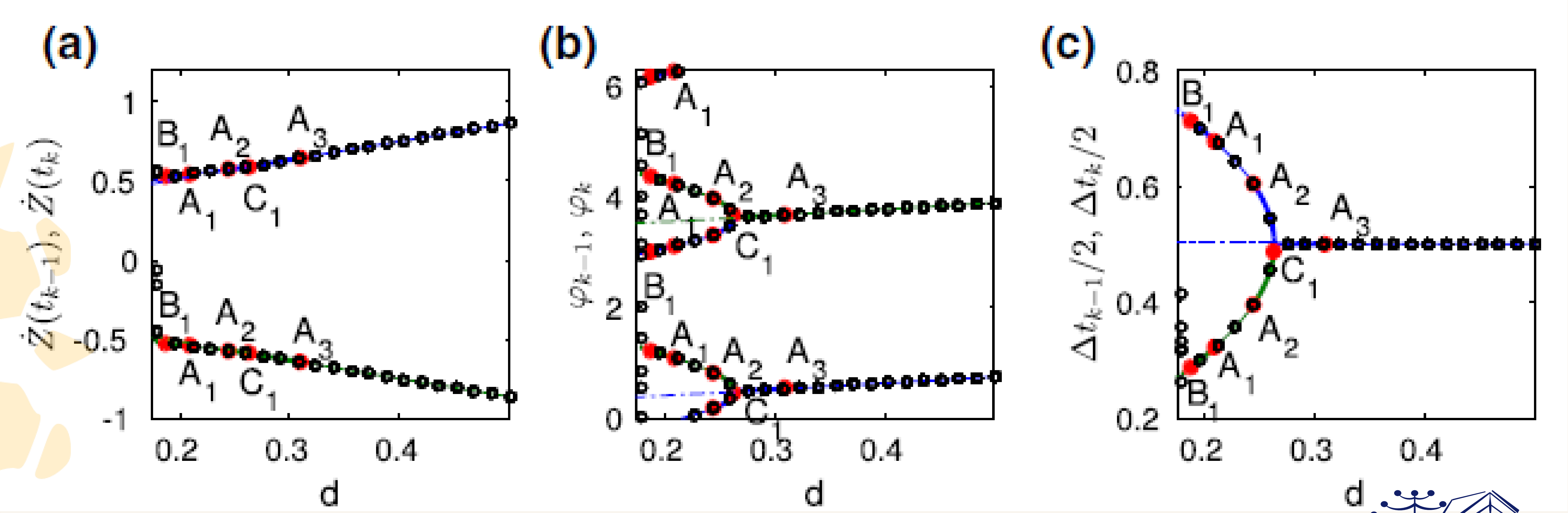
asymmetric T-periodic

$\beta = \pi/4$

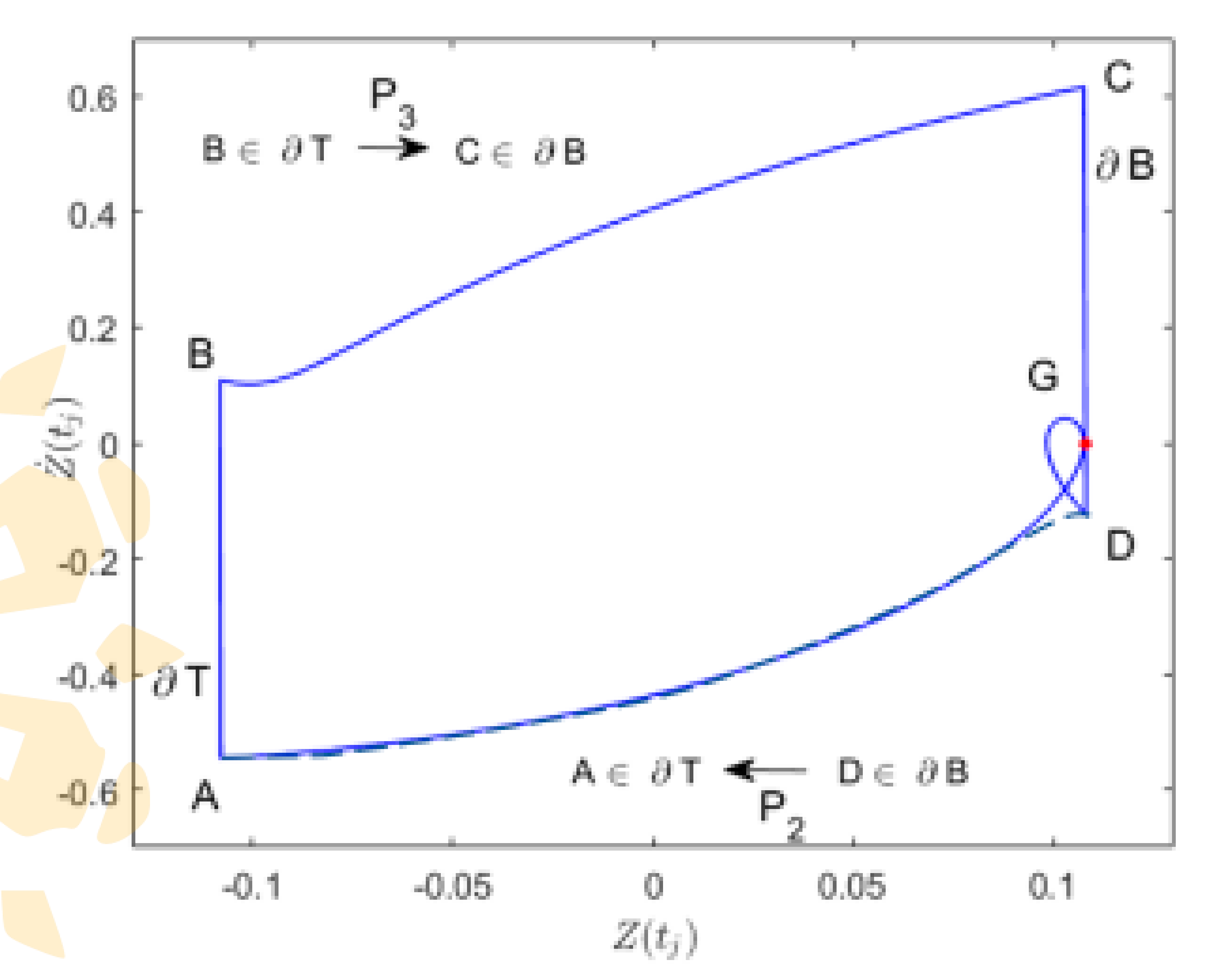
asymmetric 2T-periodic



Classical bifurcations, which can be identified from linear analysis, help us to predict motion stability



Grazing bifurcations



Using the maps and conditions

$$\dot{Z}(t_G) = 0, Z(t_G) = -d/2$$

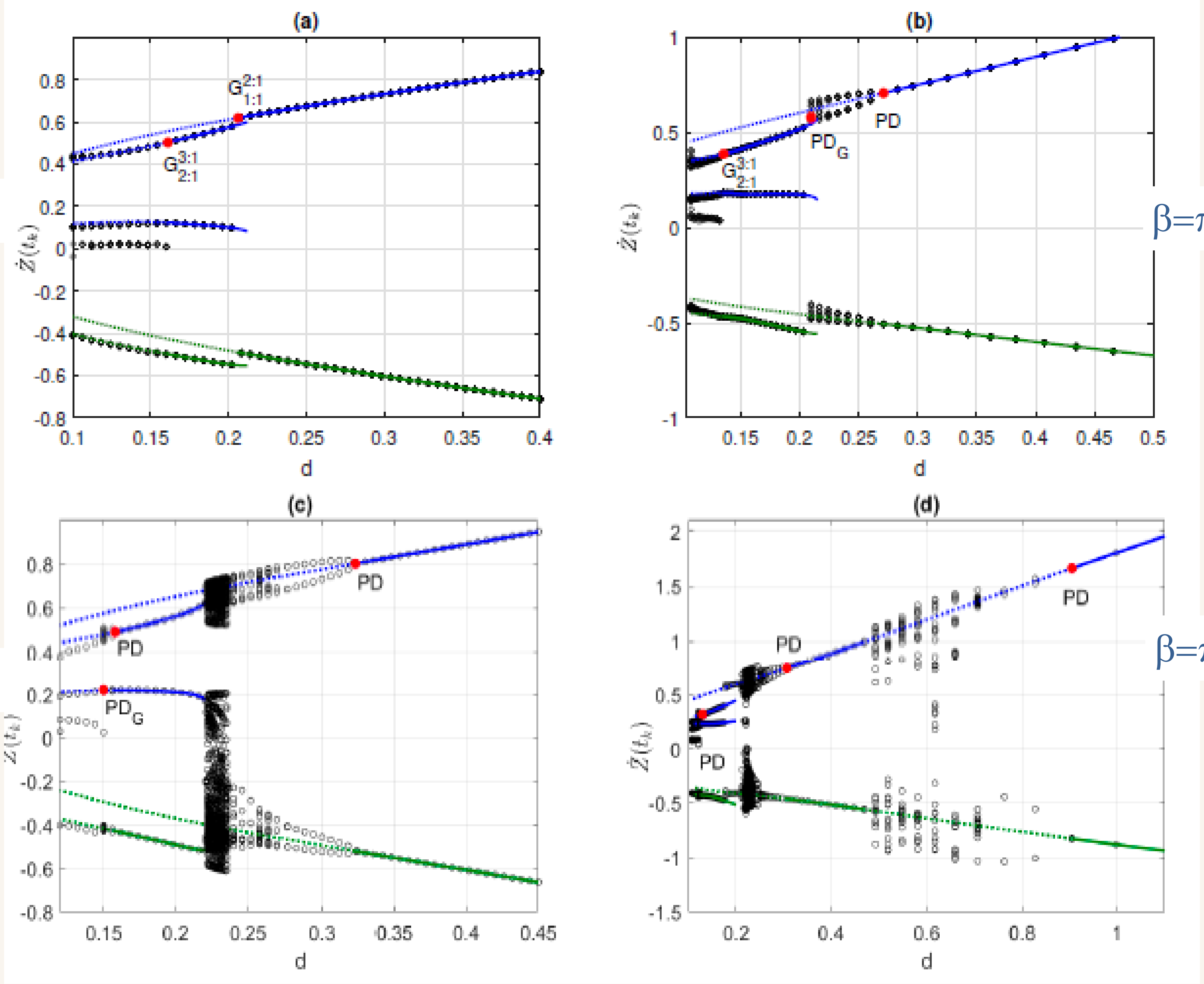
It is possible to establish the exact set of parameters for grazing.

Unlike PD and FB, the linear stability analysis cannot predict grazing bifurcation

VI-CAPSULE

β=π/9, r=0.3, A=5N

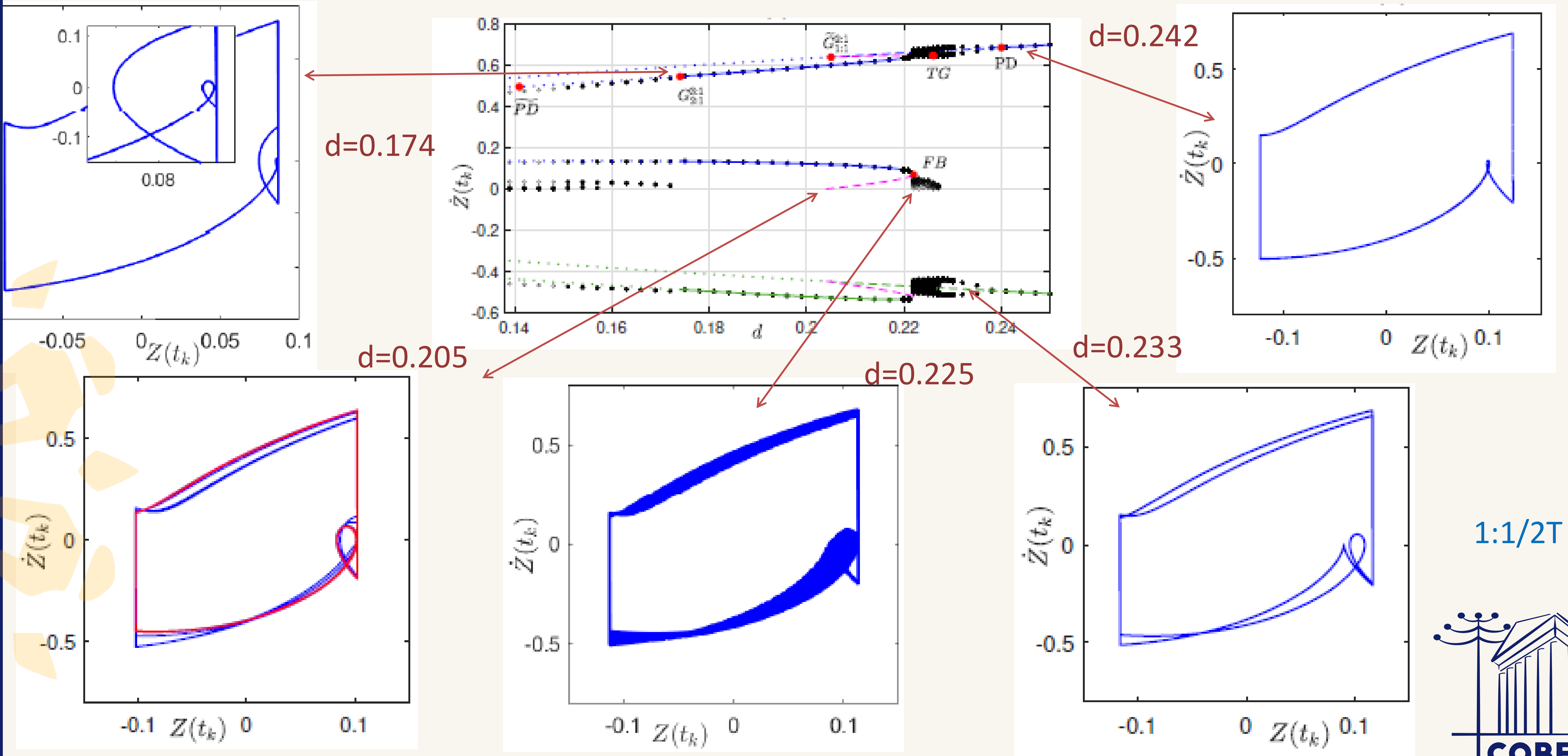
β=π/4, r=0.5, s=0.5m



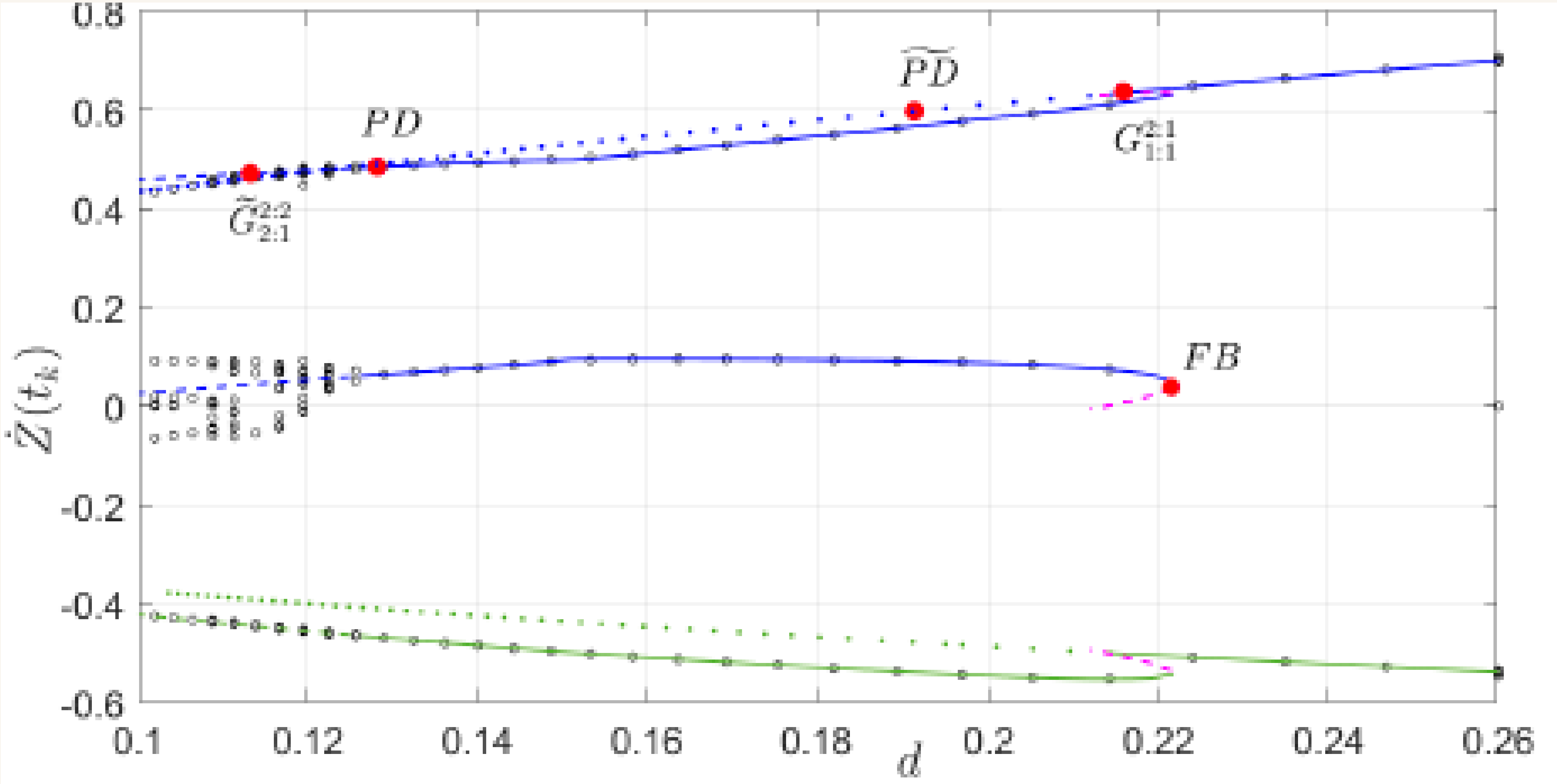
β=π/3, r=0.5, A=5N

β=π/2, r=0.7, s=0.5m

VI-CAPSULE

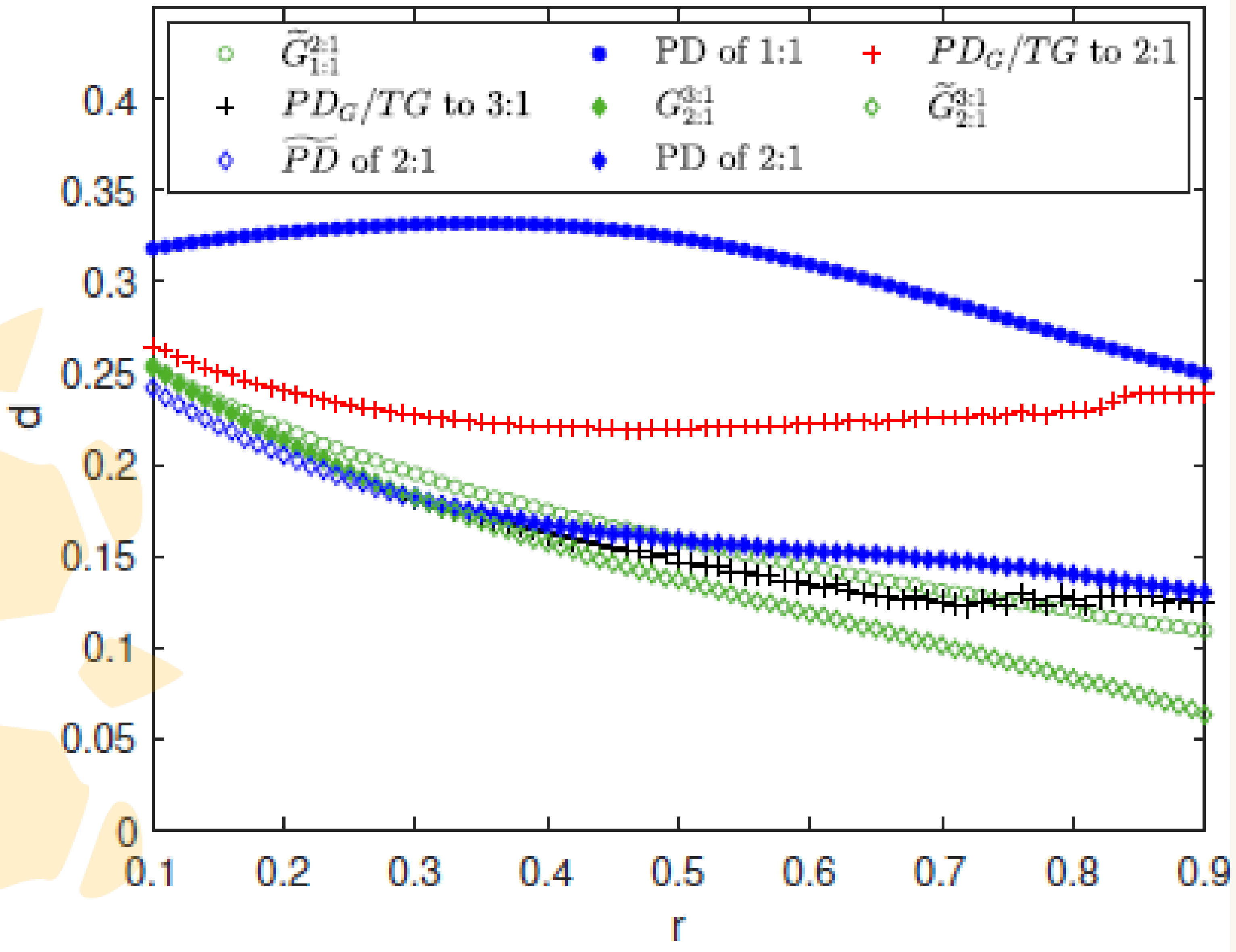

d=0.174
d=0.242
d=0.205
d=0.225
d=0.233
1:1/2T
PD
TG
FB
$\overline{PD}$
$G_{2:1}^{2:1}$
$\dot{Z}(t_k)$
$Z(t_k)$
d

VI-CAPSULE

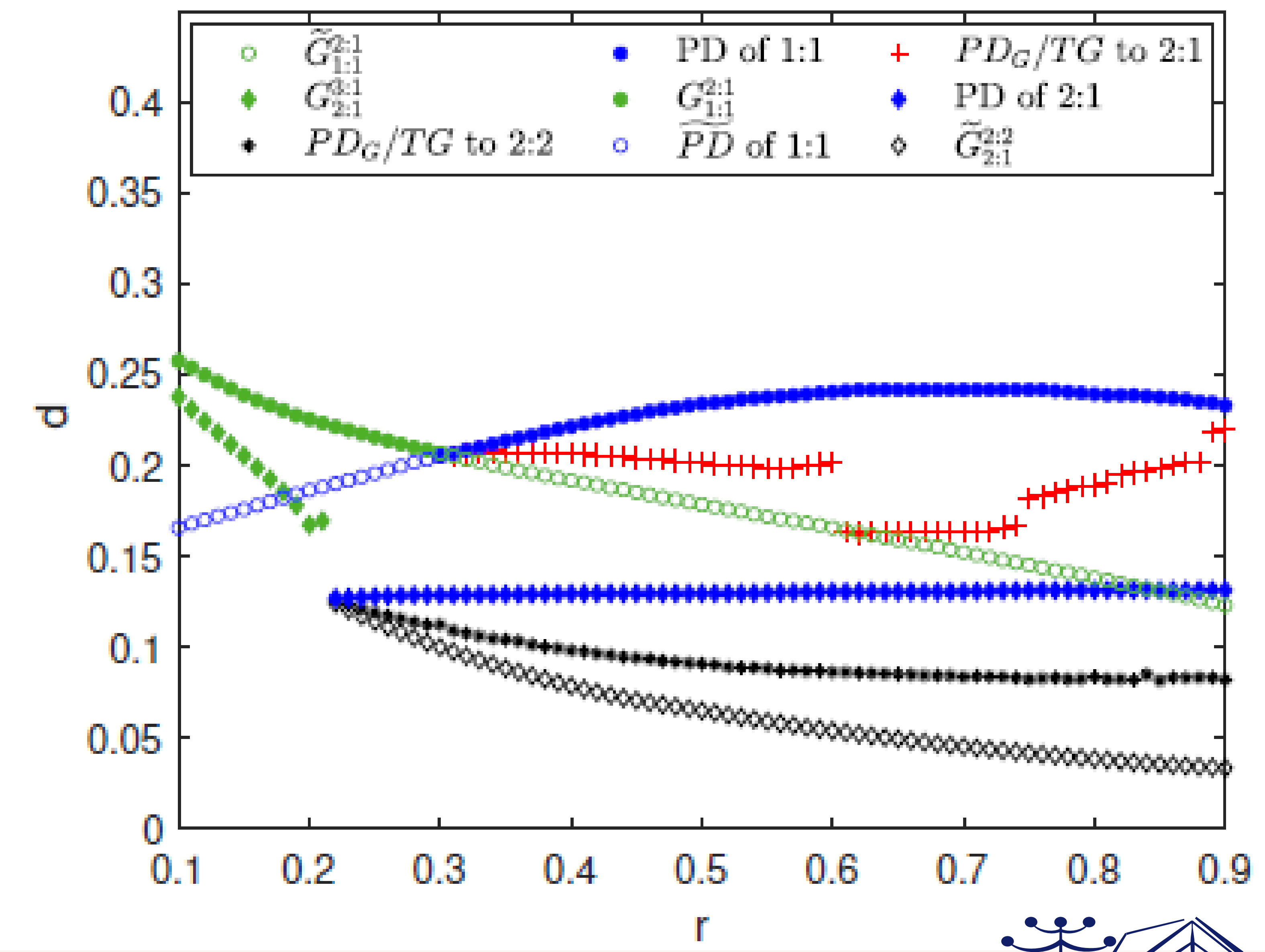

0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
d
$\dot{Z}(t_k)$
PD
$\widetilde{PD}$
$\widetilde{G}_{2:1}^{2:2}$
$G_{1:1}^{2:1}$
FB

VI-CAPSULE

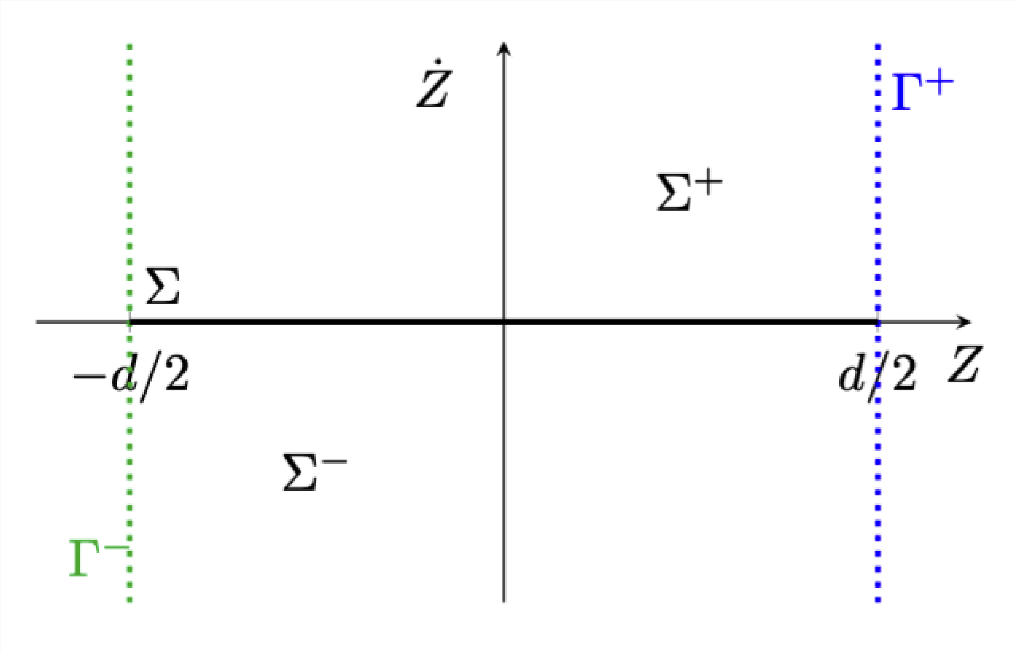
β=π/3; reducing "s"

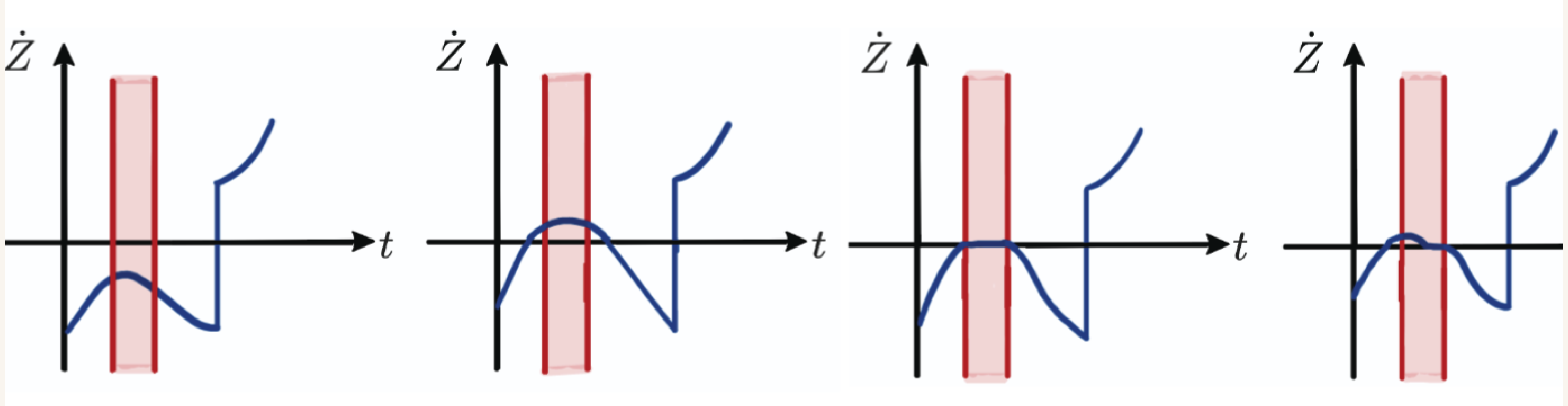


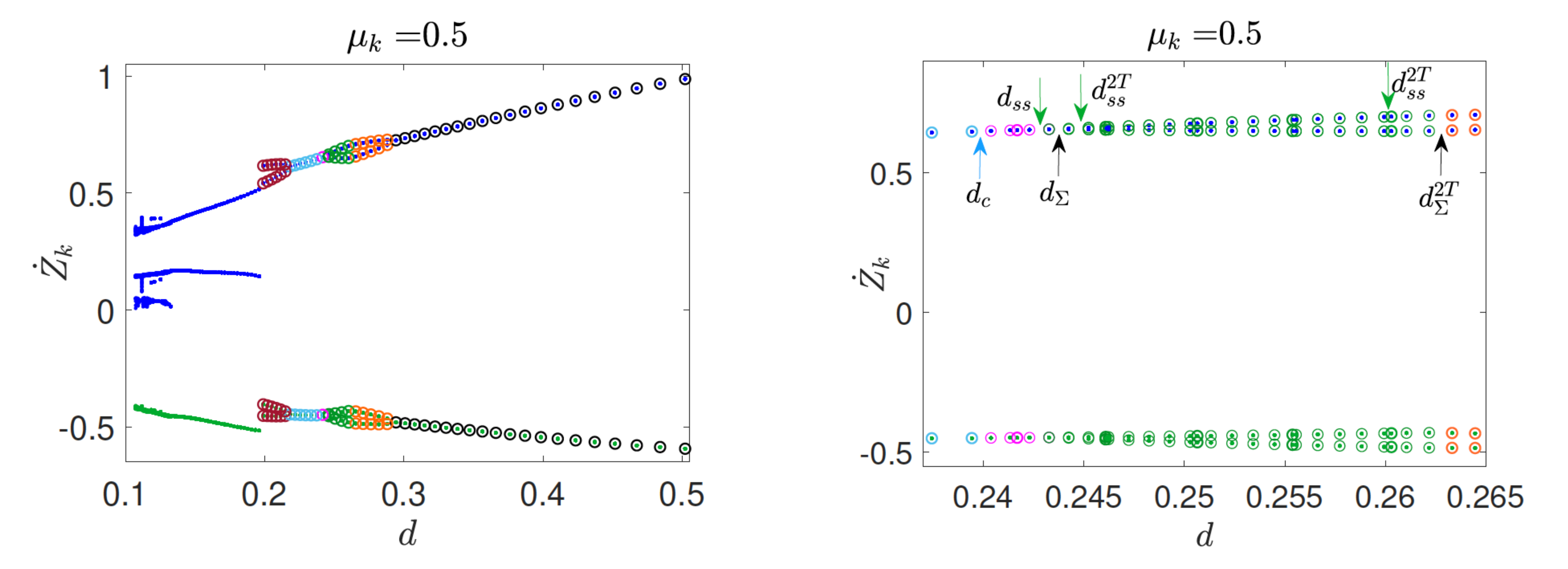
β=π/6; increasing "A"



VI-CAPSULE with FRICTION



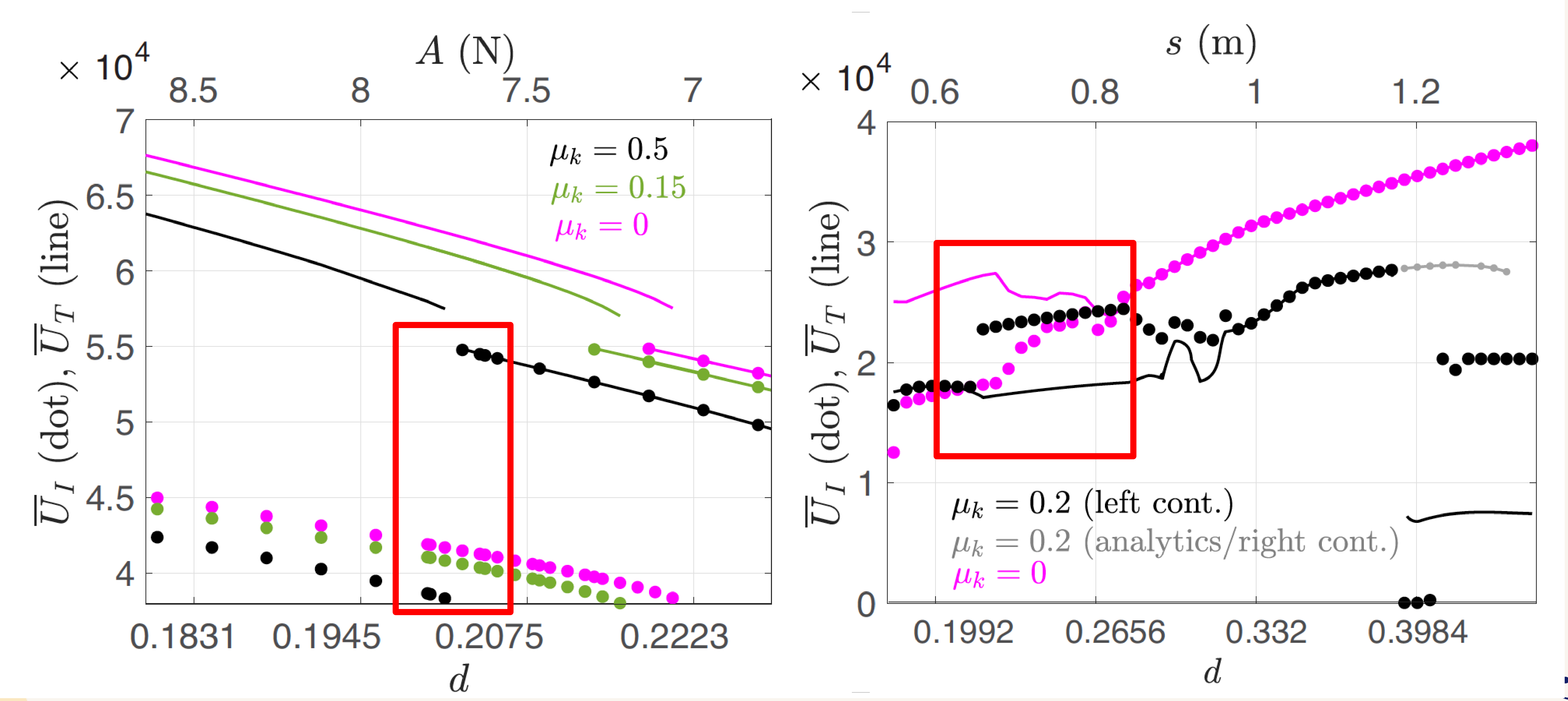




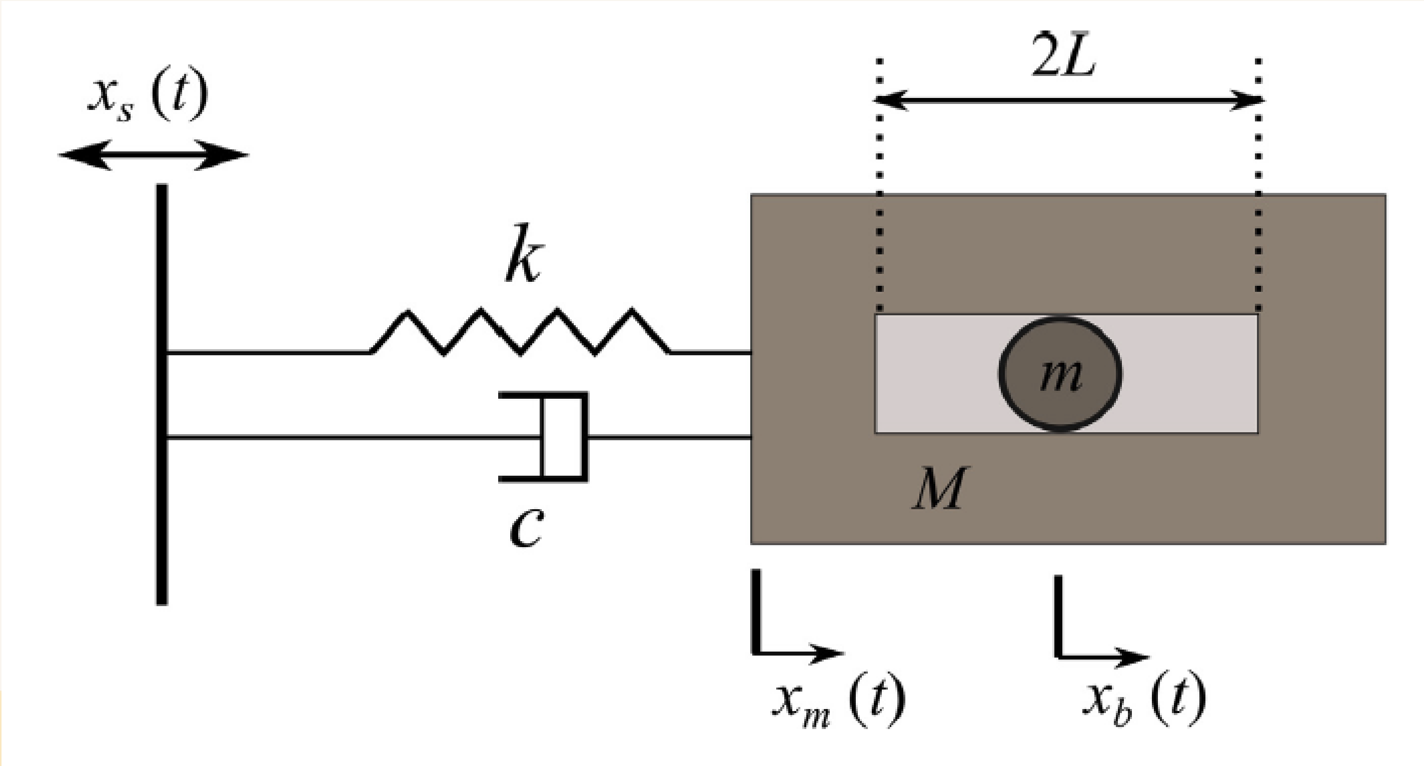
$$1{:}1 \xrightarrow{PD} 1{:}1/2T \xrightarrow{d_\Sigma^{2T}} 1{:}1\text{-}1{:}1_s^\Sigma/2T \xrightarrow{d_{ss}^{2T}} 1{:}1\text{-}1{:}1_{cs}^\Sigma/2T \xrightarrow{d_{ss}^{2T}} 1{:}1\text{-}1{:}1_s^\Sigma/2T \xrightarrow{d_\Sigma}$$

$$1{:}1_s^\Sigma \xrightarrow{d_{ss}} 1{:}1_{cs}^\Sigma \xrightarrow{d_c} 1{:}1_c^\Sigma \xrightarrow{d_c^{2T}} 1{:}1_c^\Sigma/2T \xrightarrow{d_{\Gamma+}} 2{:}1\,,$$

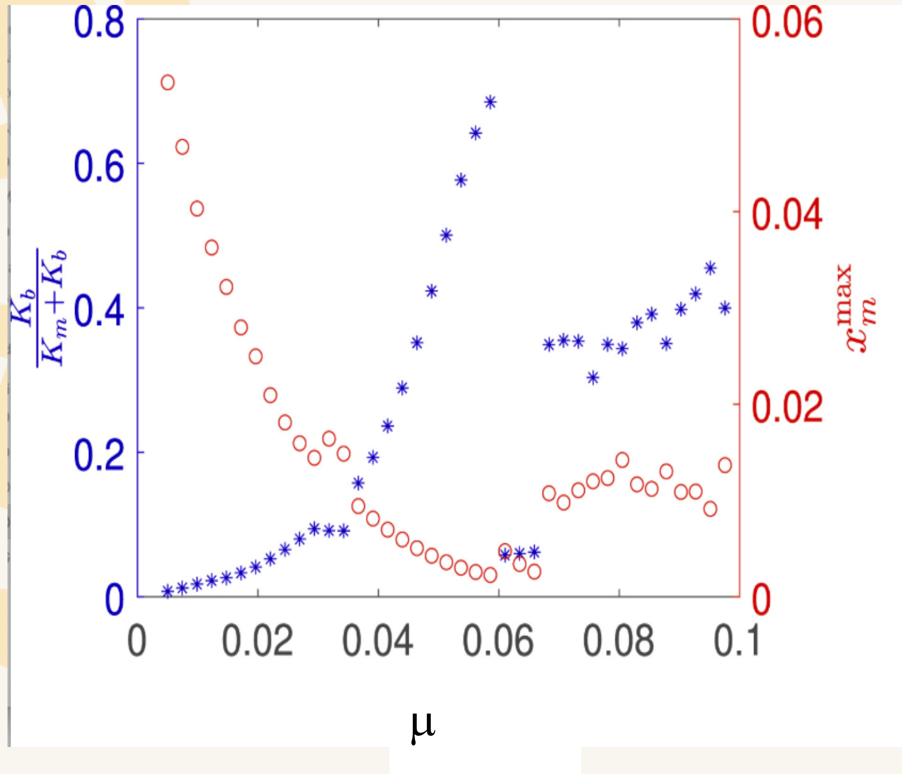
VI-CAPSULE with FRICTION



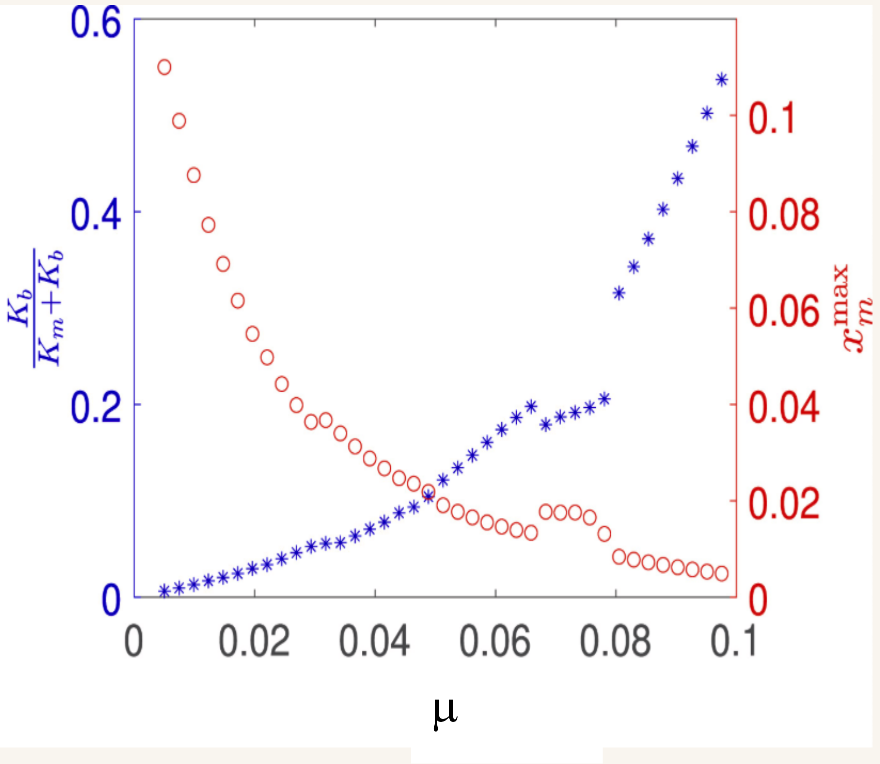
Targeted Energy Transfer

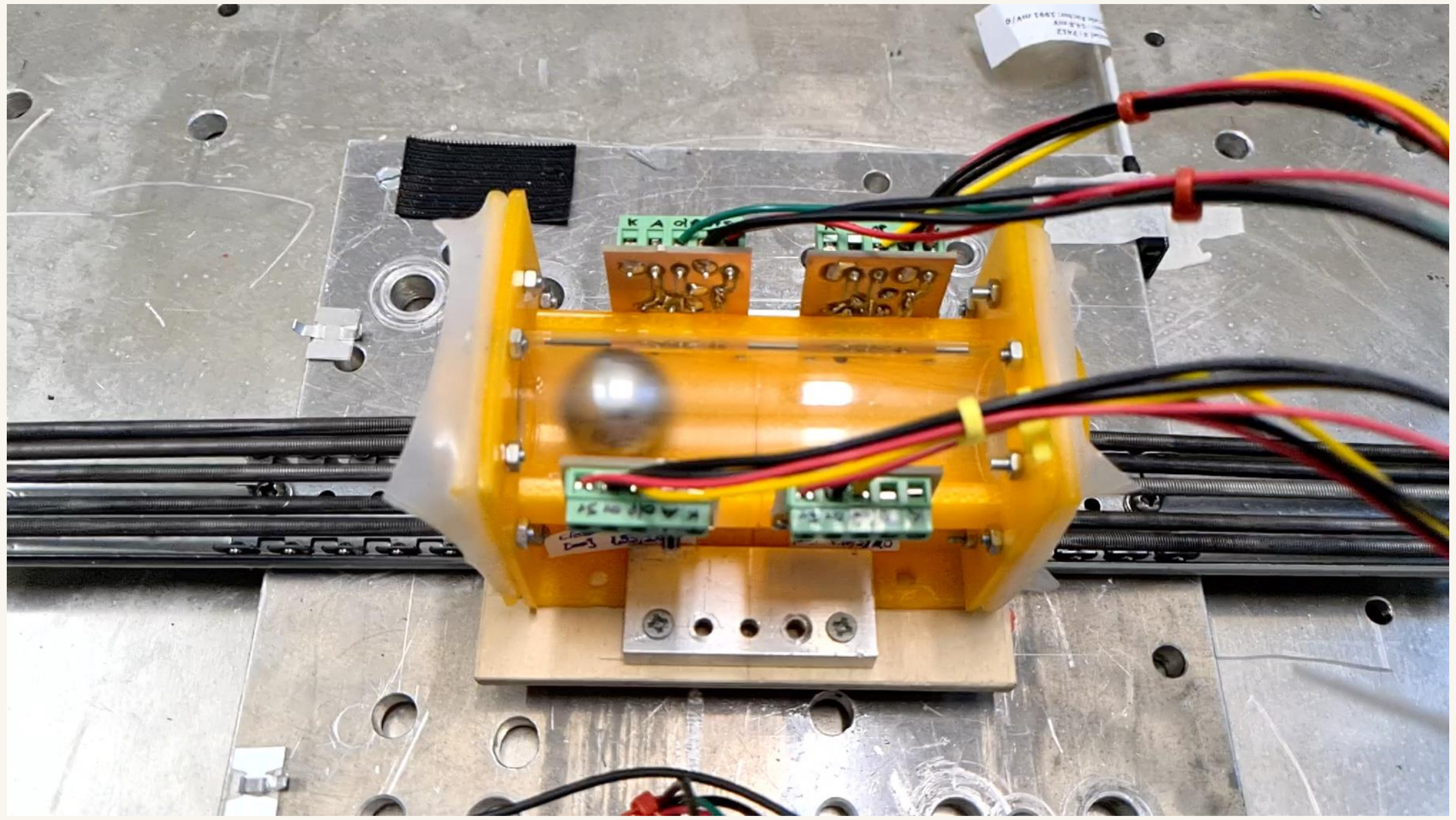


F=0.05

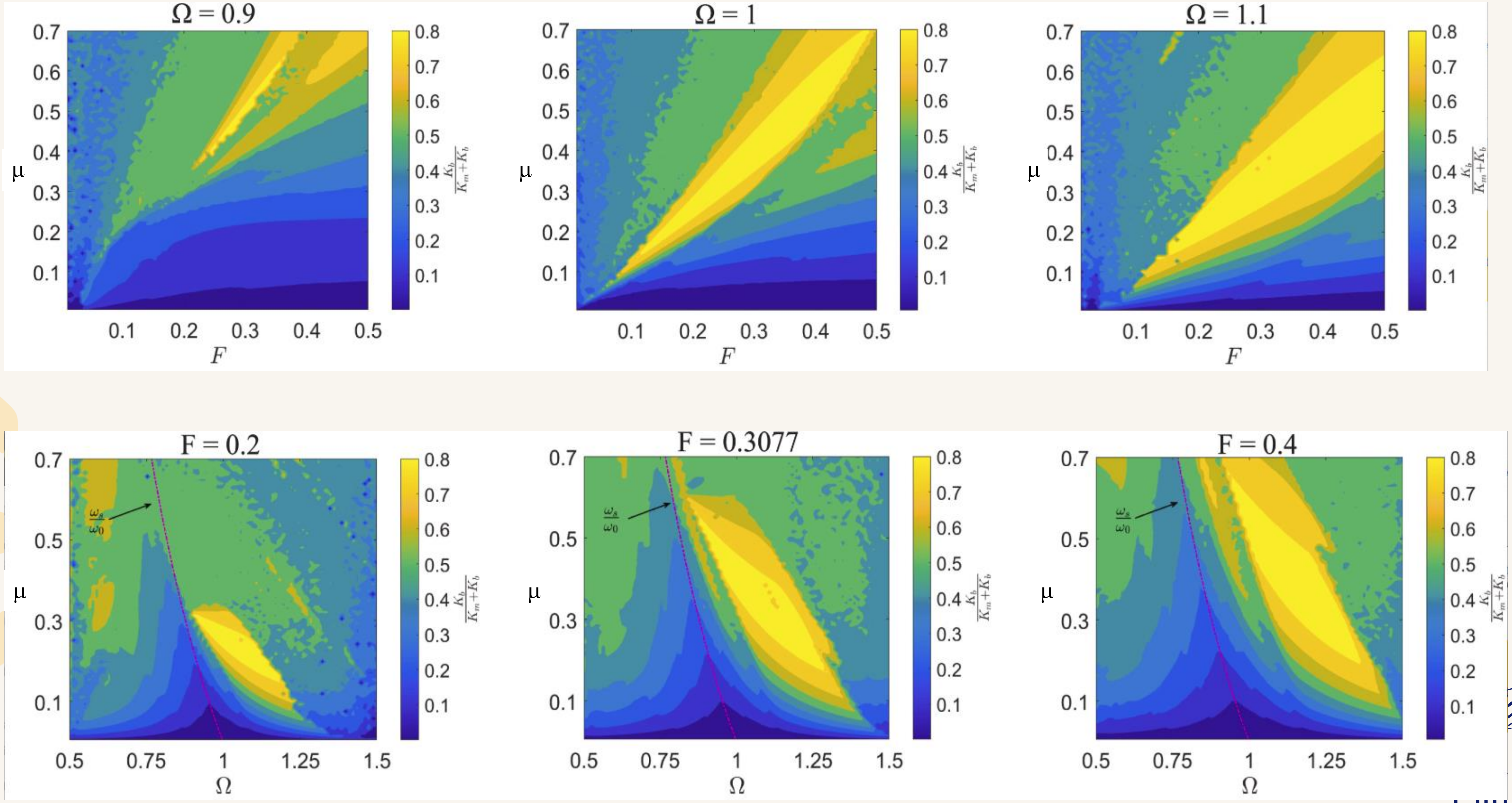


F=0.1





Targeted Energy Transfer



Computer-assisted method

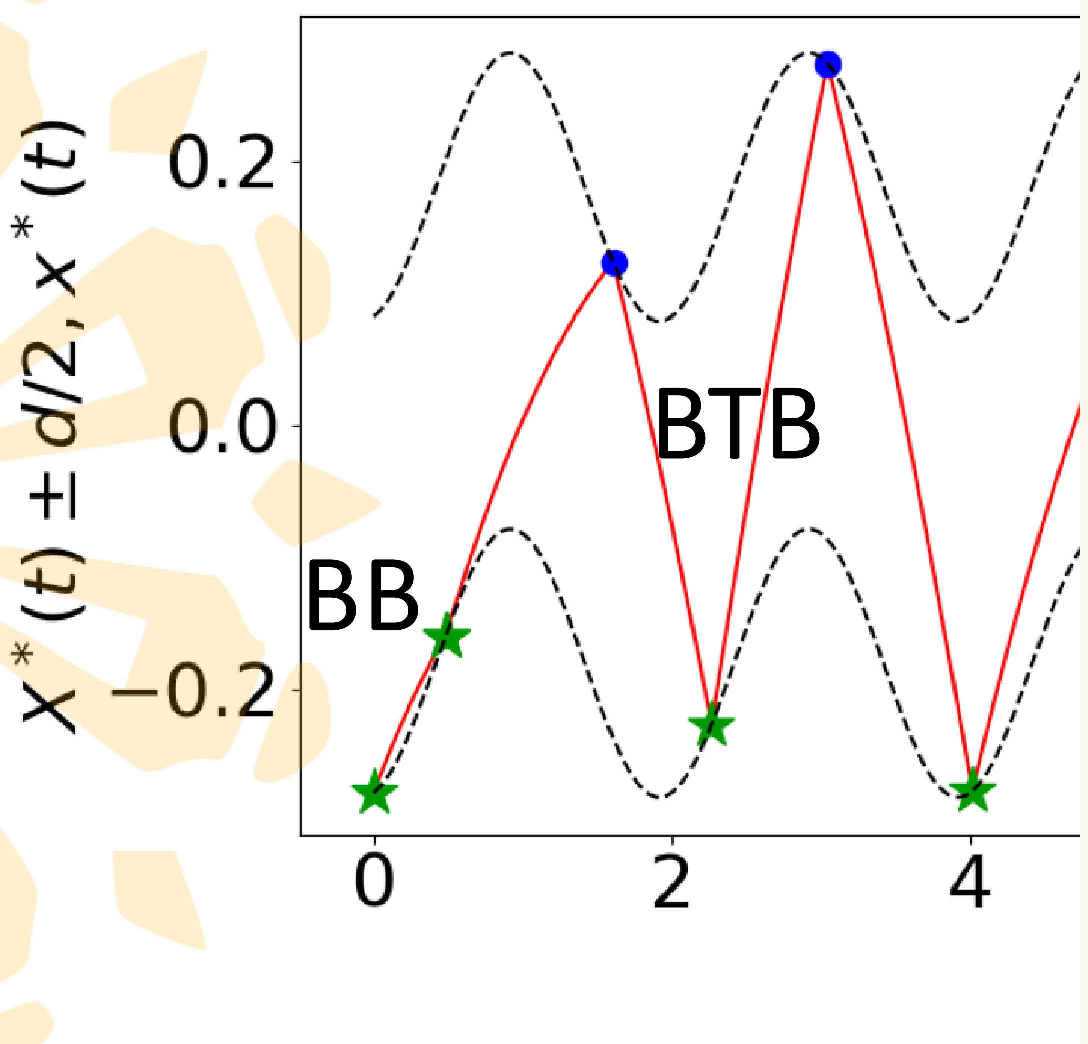
First return maps:

Use the short sequences of returns to ∂B return maps as the building blocks

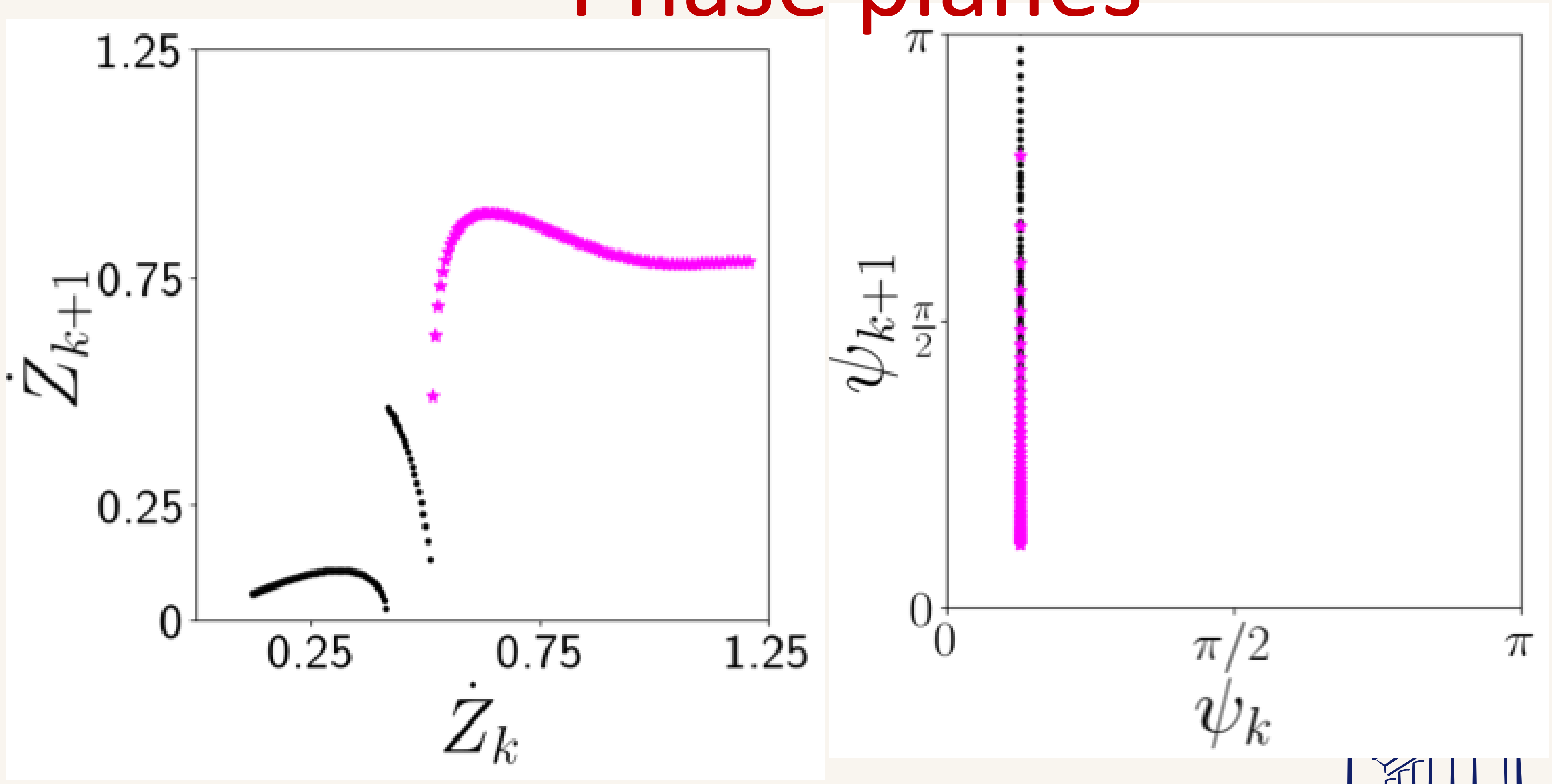
Over state space:

Sweep $\dot{Z}_k$ fixed` ψ_k

Time series



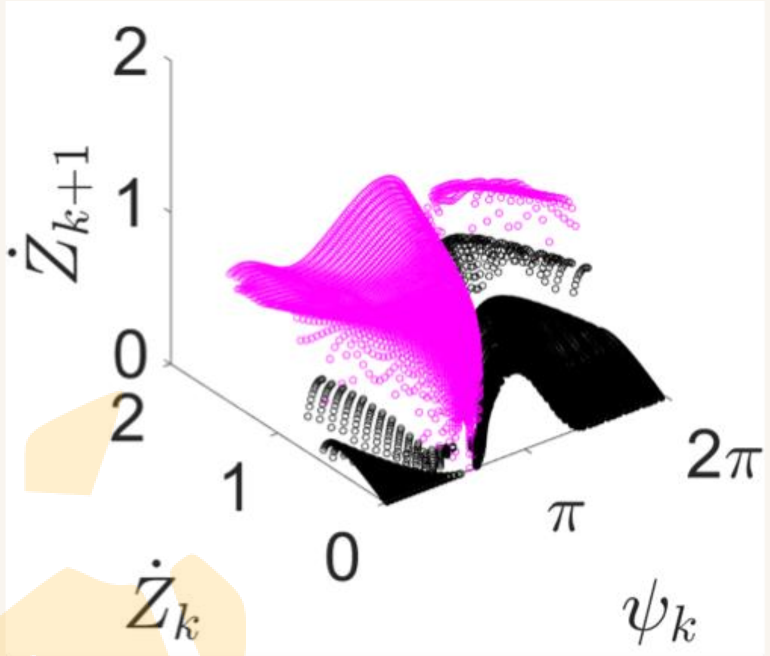
Phase planes



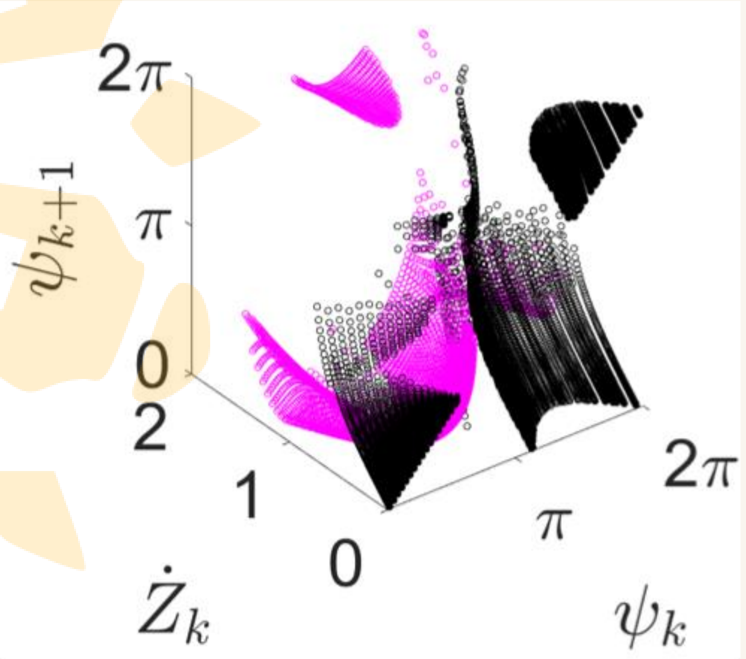
Computer-assisted method

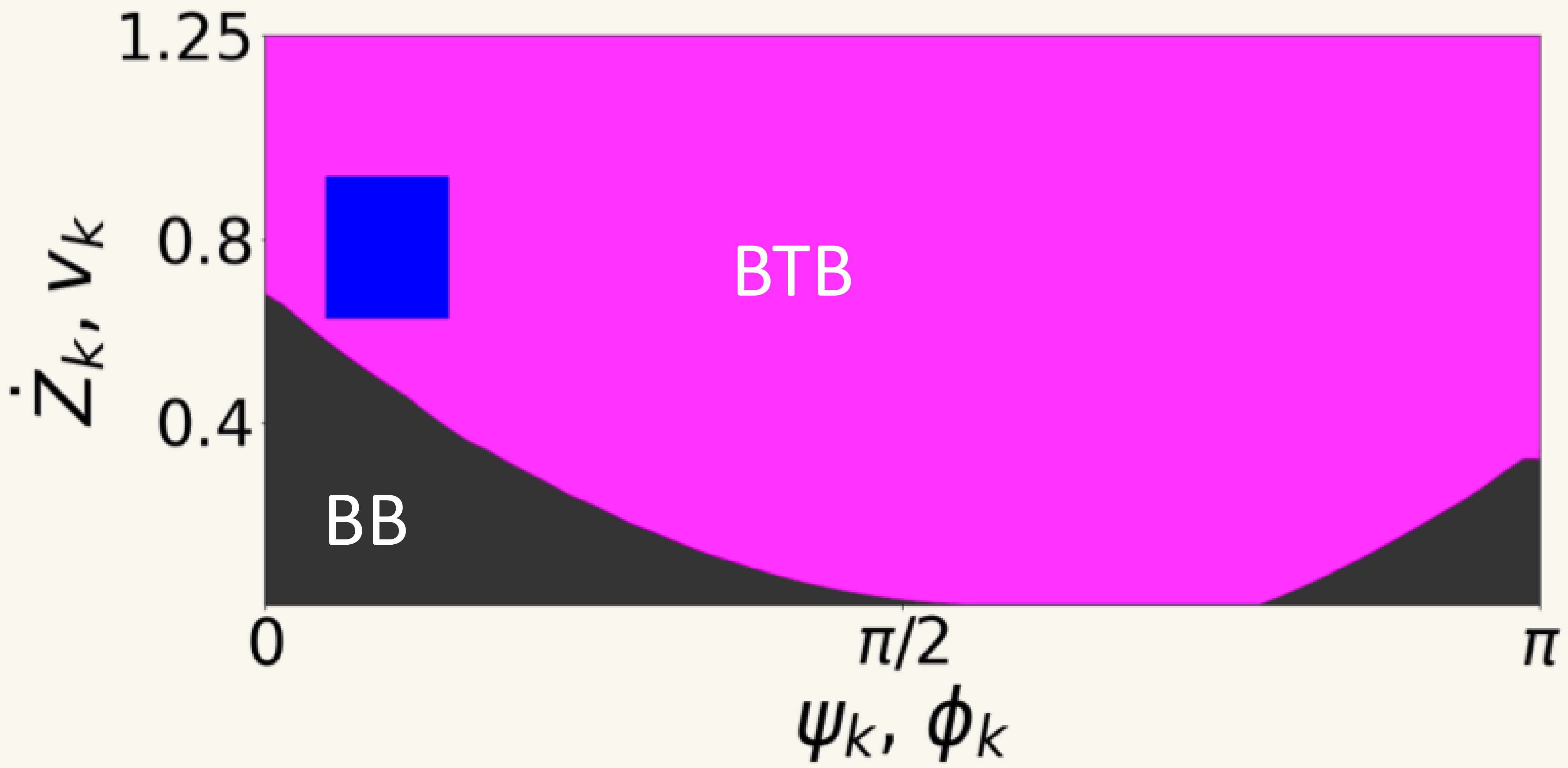
Return maps in Relative framework:

Impact velocity

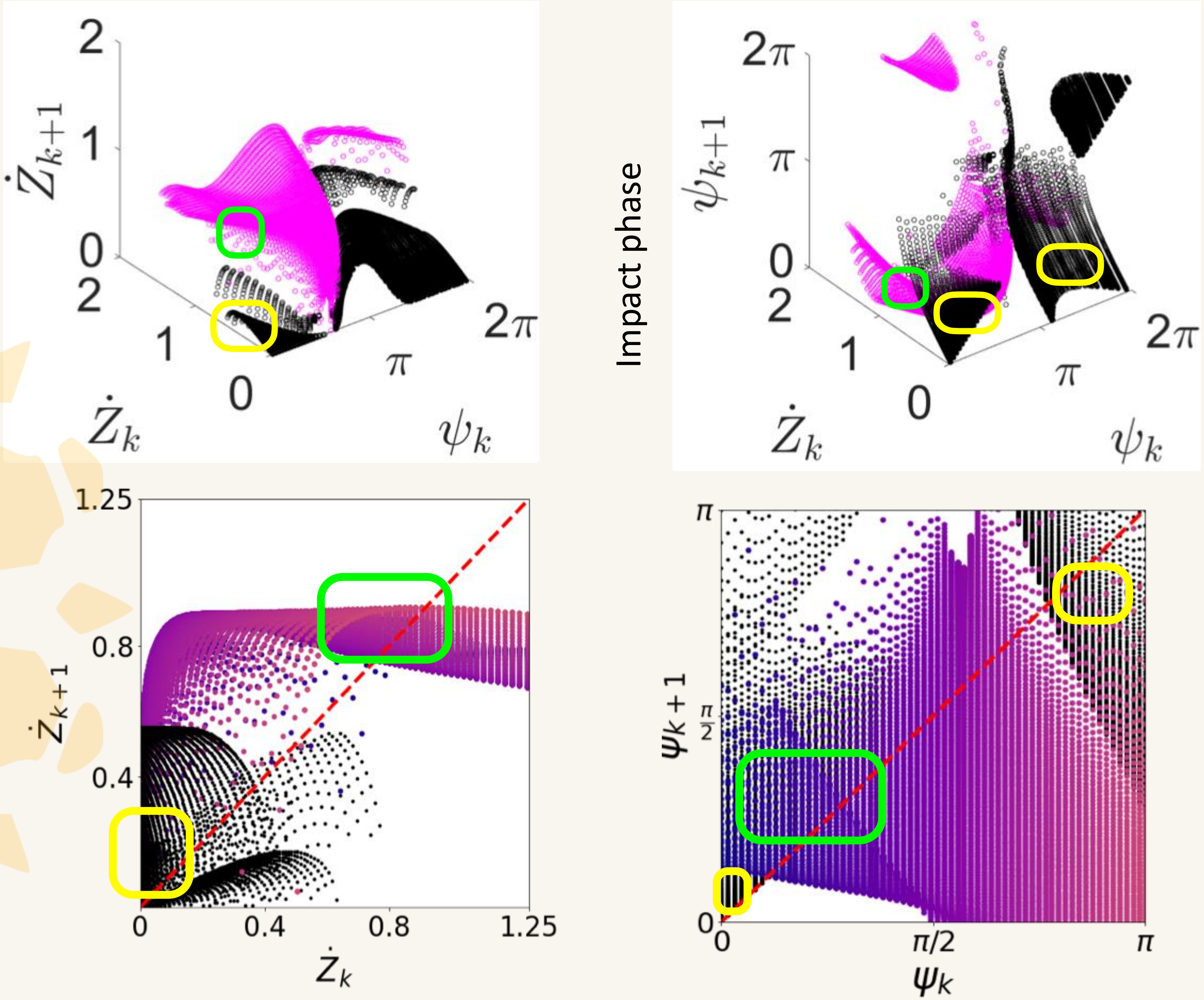


Impact phase





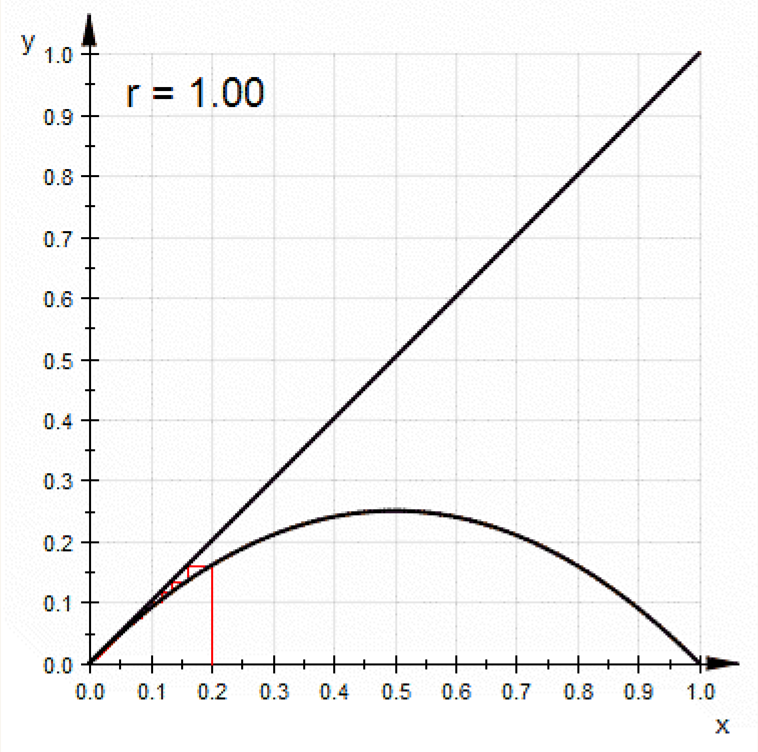
Computer-assisted method



Exchange non-smooth impacts for non-smooth surfaces

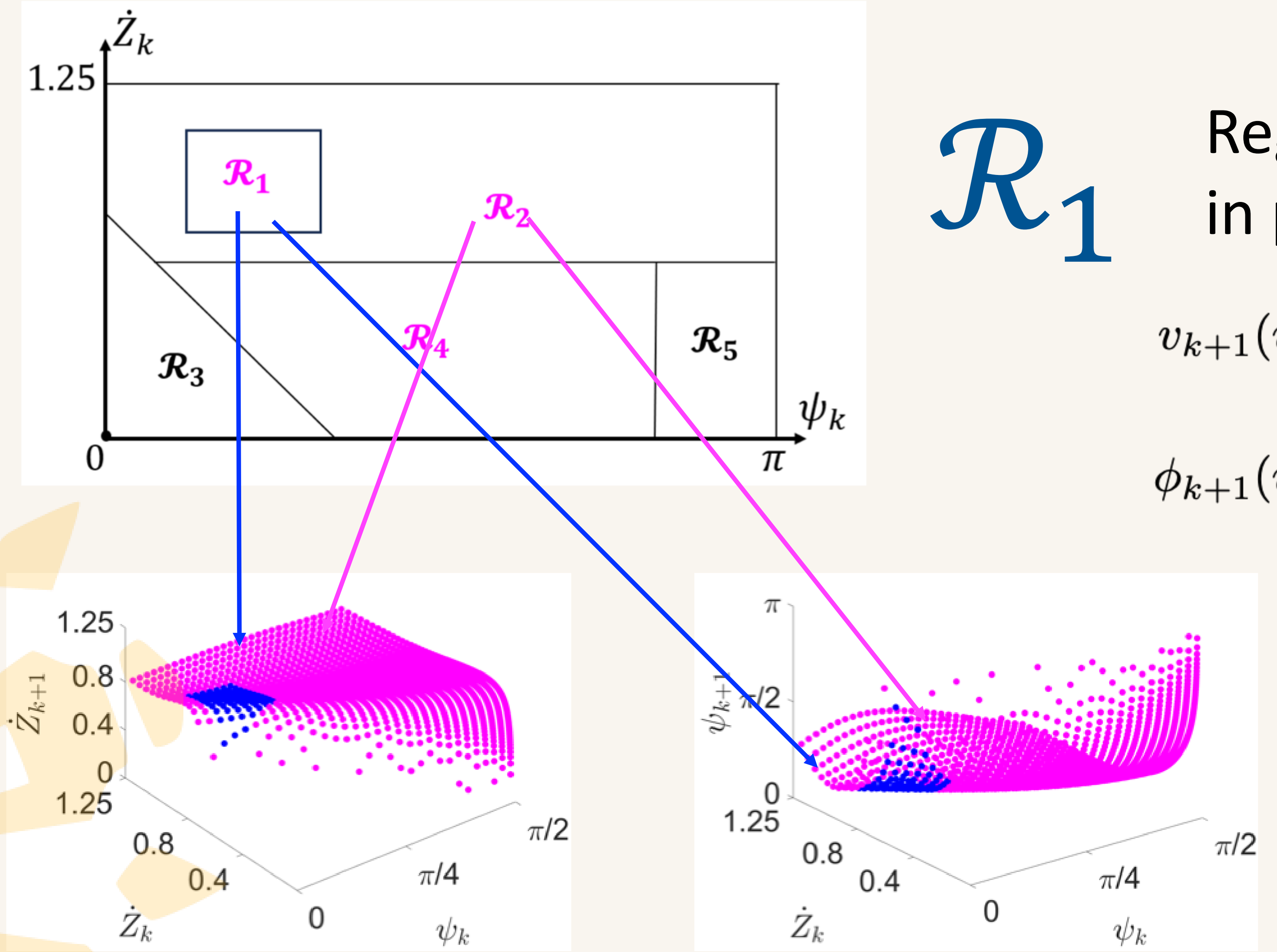
potential attracting dynamics

potential transient dynamics



Cobweb
Logic map

Computer-assisted method



$\mathcal{R}_1$ Region of likely return to same values of $\dot{Z}_k$, ψ_k in phase plane, return near the diagonal

$$v_{k+1}(v_k,\phi_k) = f_1(v_k,\phi_k) = b_0 + b_1\phi_k + b_2 v_k + b_3\phi_k^2 + b_4\phi_k v_k + b_5 v_k^2 + b_6\phi_k^2 v_k + b_7\phi_k v_k^2 + b_8 v_k^3,$$

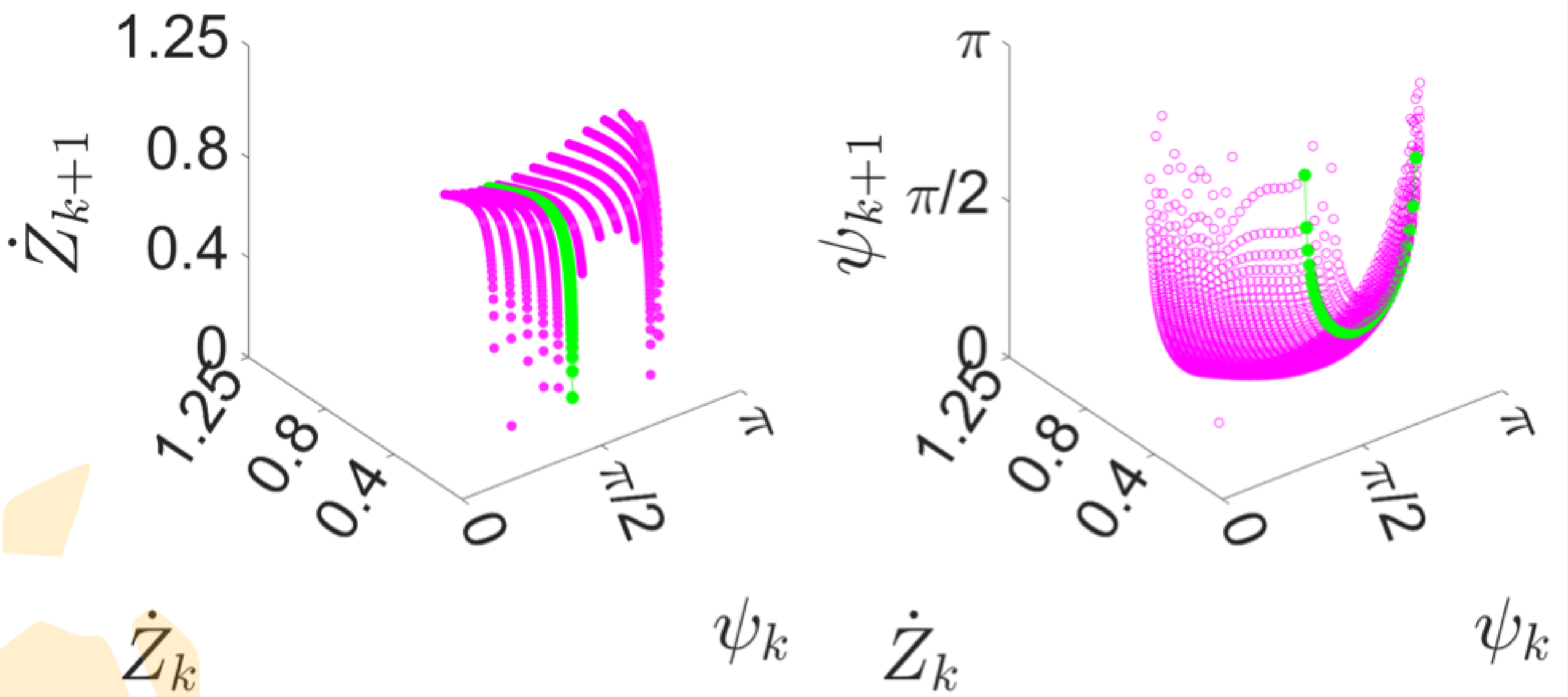
$$\phi_{k+1}(v_k,\phi_k) = g_1(v_k,\phi_k) = a_0 + a_1\phi_k + a_2 v_k + a_3\phi_k^2 + a_4\phi_k v_k + a_5 v_k^2 + a_6\phi_k^2 v_k + a_7\phi_k v_k^2 + a_8 v_k^3.$$

$$\mathcal{R}_{1.2}(d) = \left\{(\dot{Z}_k,\psi_k)\ :\ \frac{1}{\delta} < \left|\frac{\psi_{k+1}}{\psi_k}\right| < \delta \text{ and } \frac{1}{\delta} < \left|\frac{\dot{Z}_{k+1}}{\dot{Z}_k}\right| < \delta\ \right\}$$

Remainder of the region is transient BTB behavior: shape of surface and/or not near diagonal in phase space

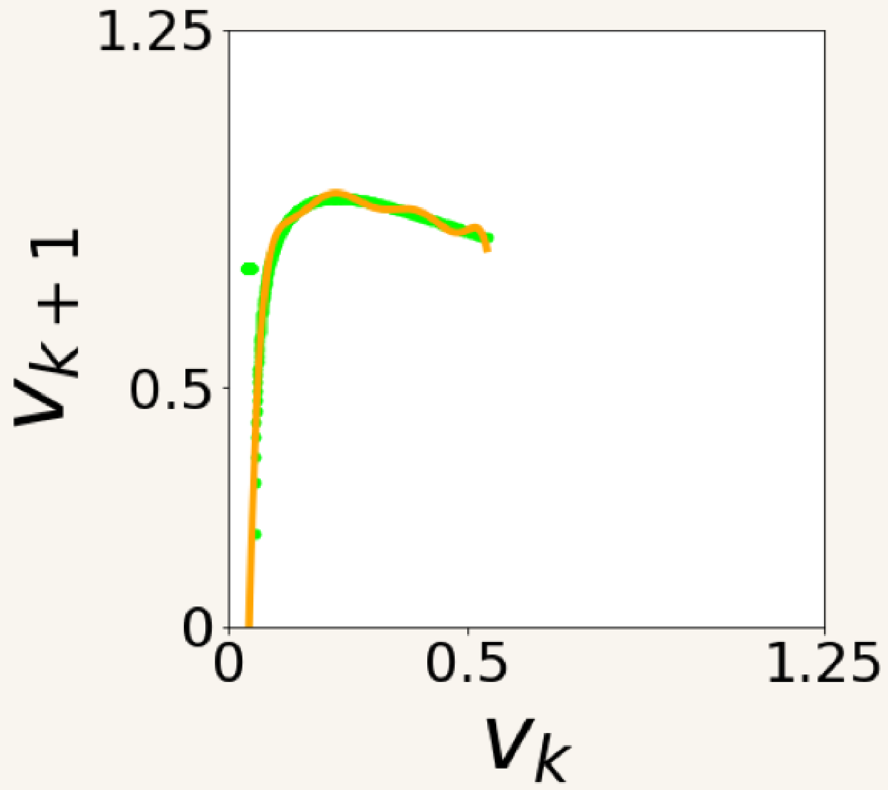
Computer-assisted method

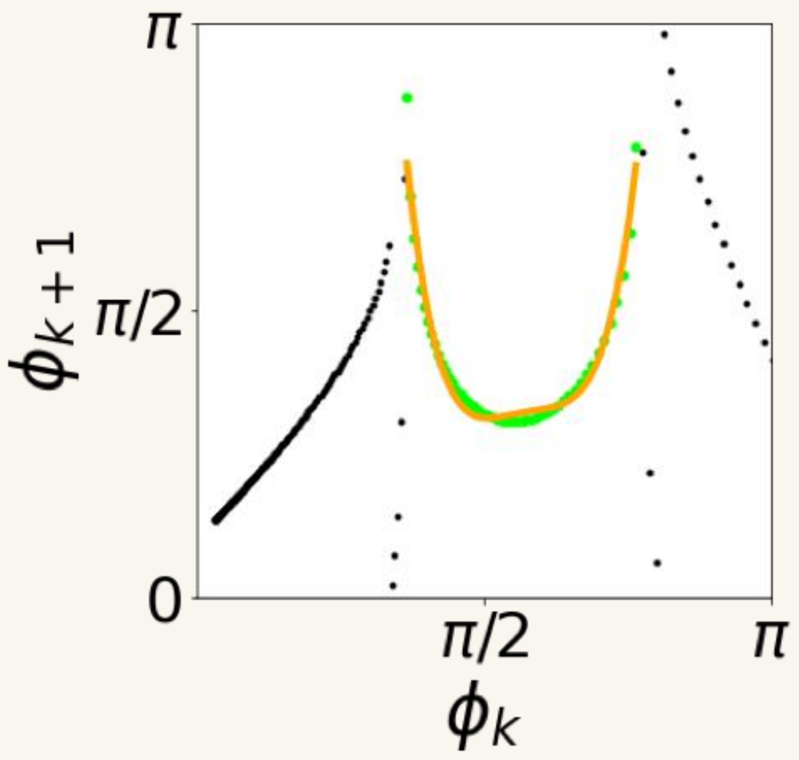
Approximate separable 1D maps in transient regions (R2, R4, R5):



$$v_{k+1}(v_k) = f_2(v_k) = b_{20}v_k^5 + b_{21}v_k^4 + b_{22}v_k^3 + b_{23}v_k^2 + b_{24}v_k + b_{25},$$

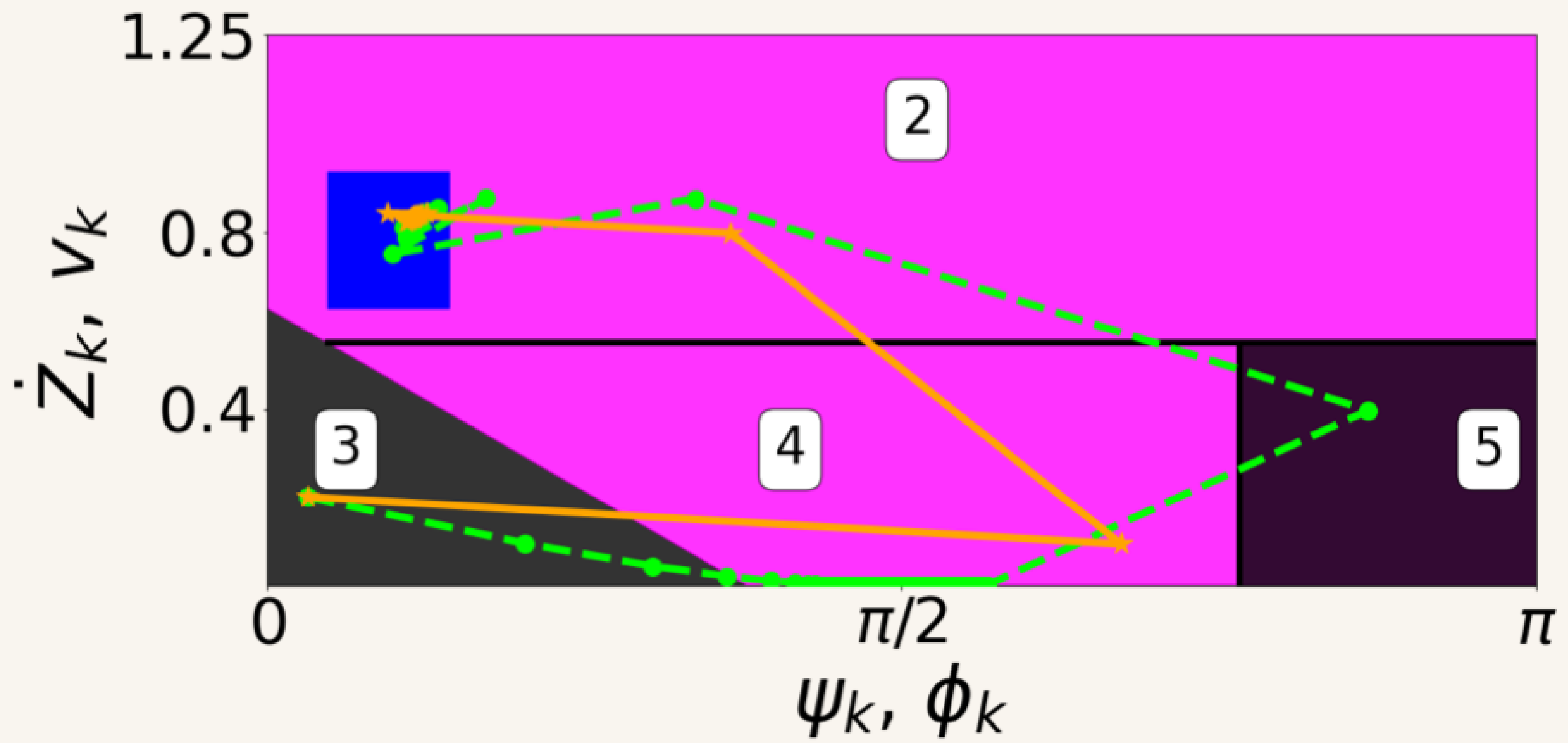
$$\phi_{k+1}(\phi_k) = g_2(\phi_k) = a_{20}\phi_k^5 + a_{21}\phi_k^4 + a_{22}\phi_k^3 + a_{23}\phi_k^2 + a_{24}\phi_k + a_{25}.$$



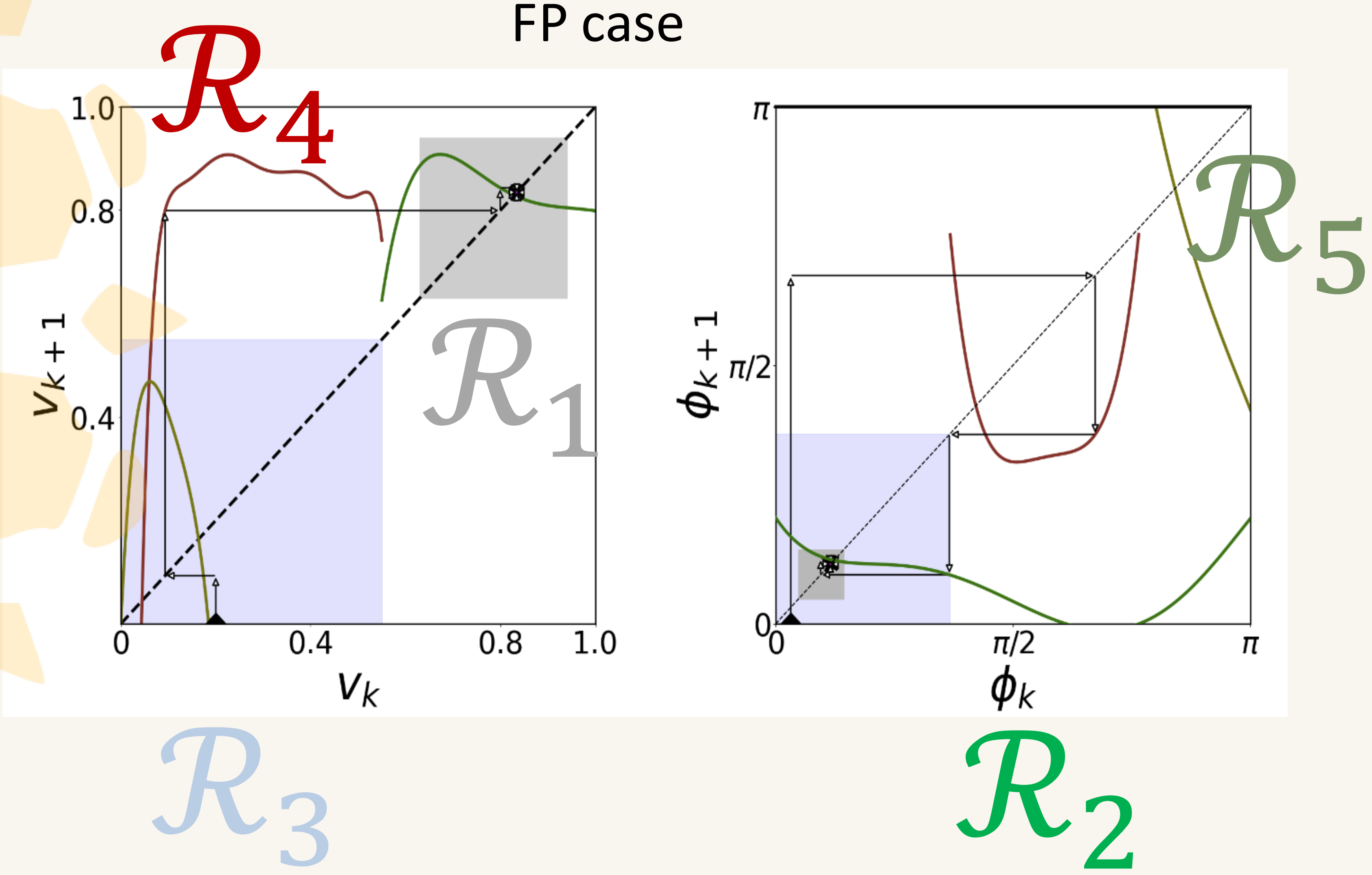


COBEM 2025

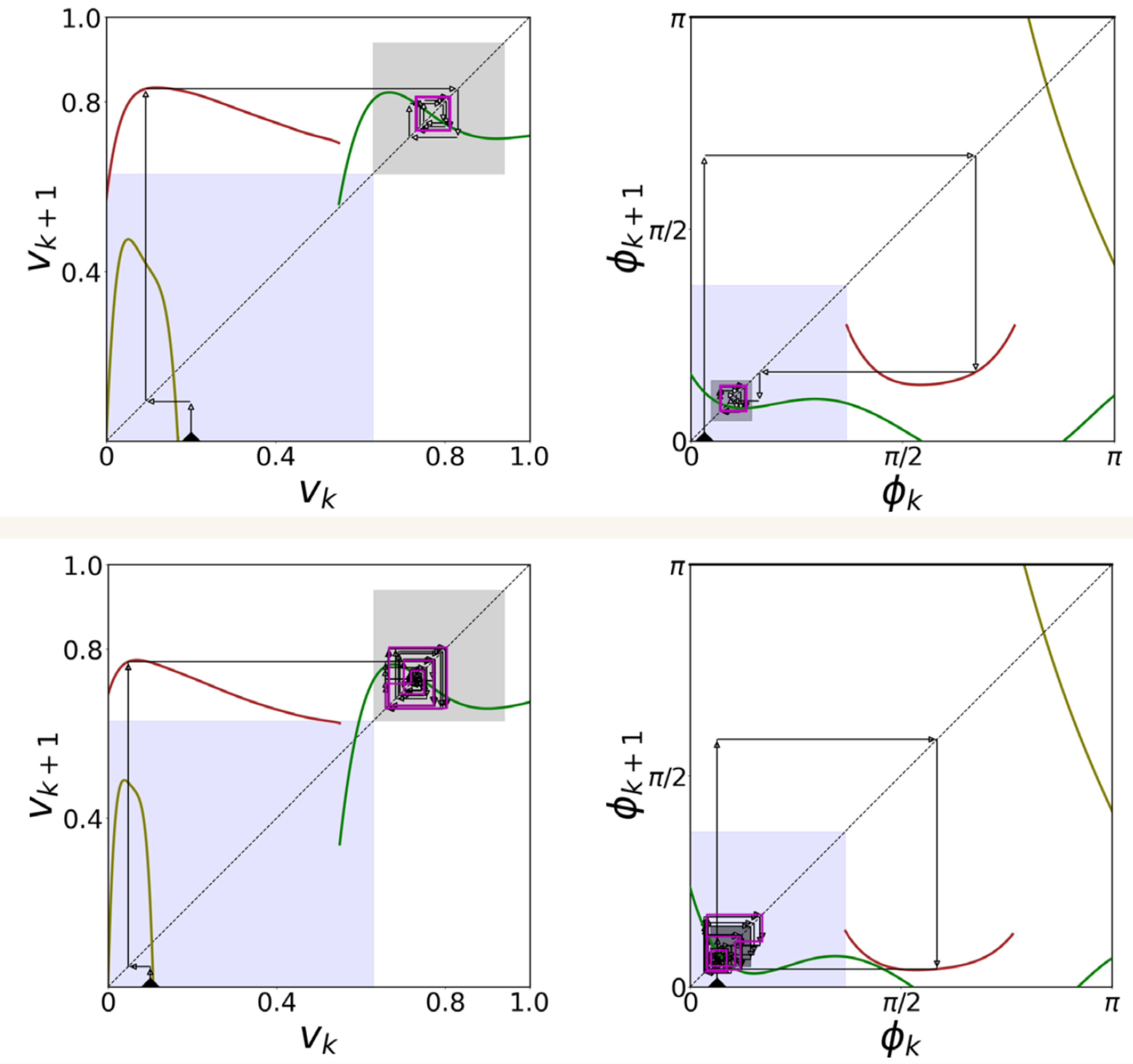
Computer-assisted method



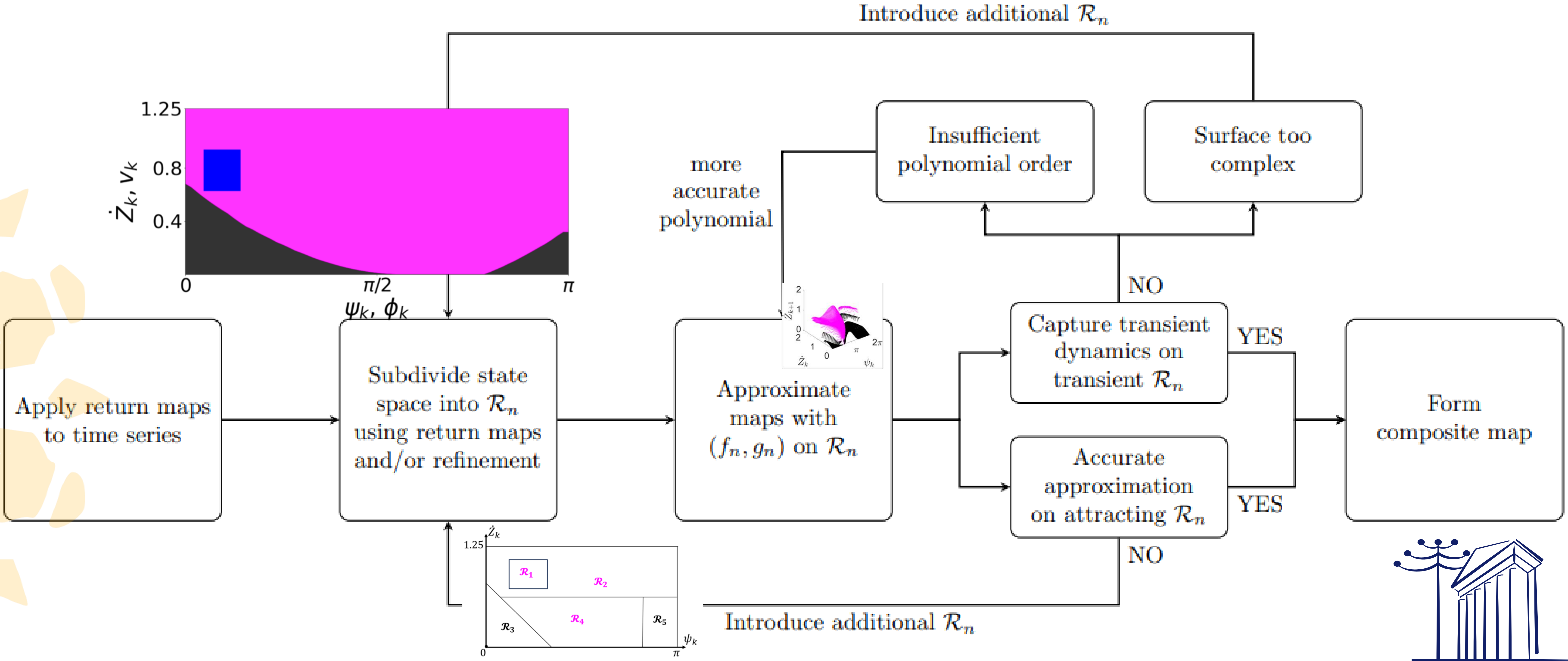
FP case



Computer-assisted method



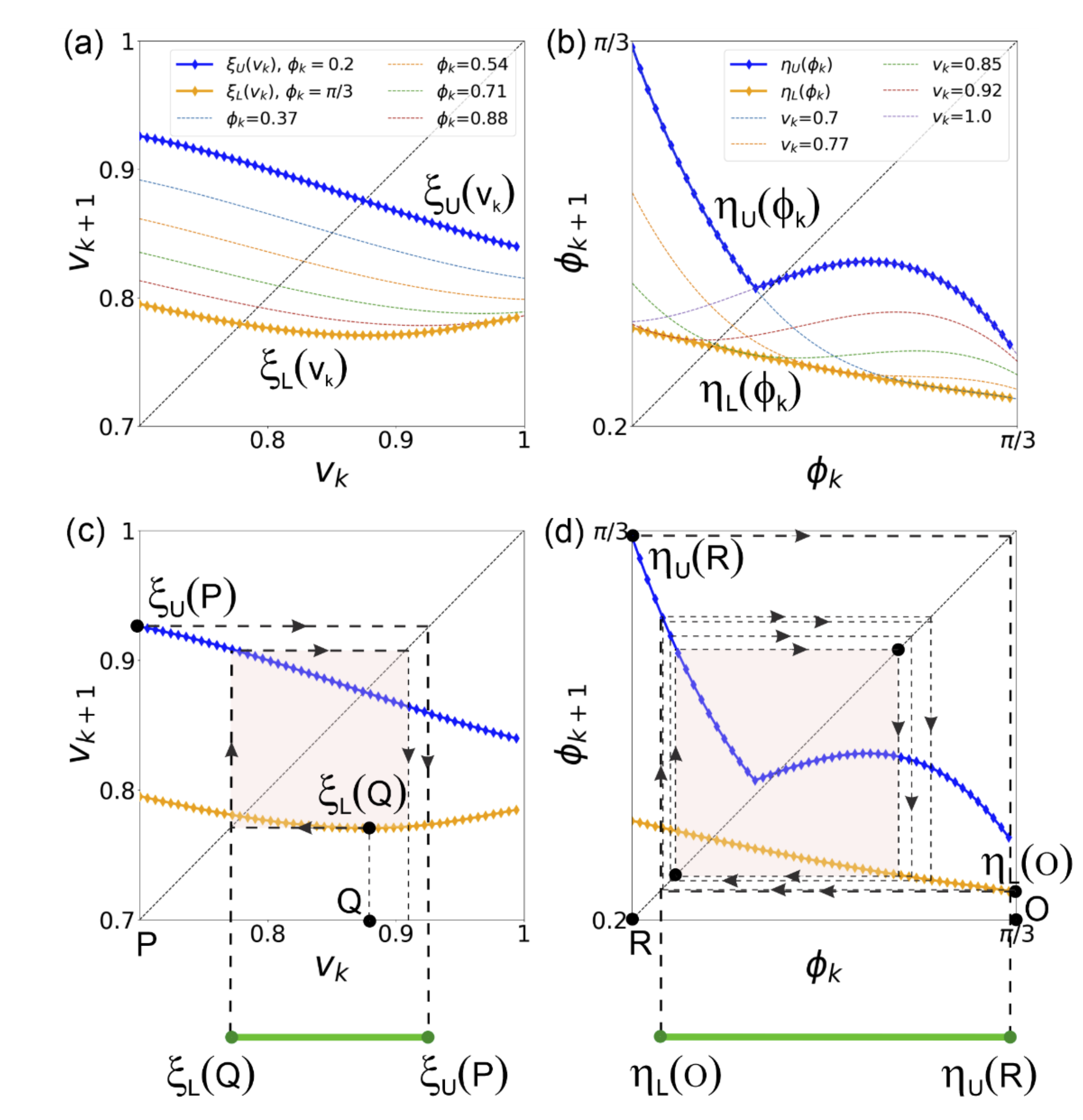
Computer-assisted method

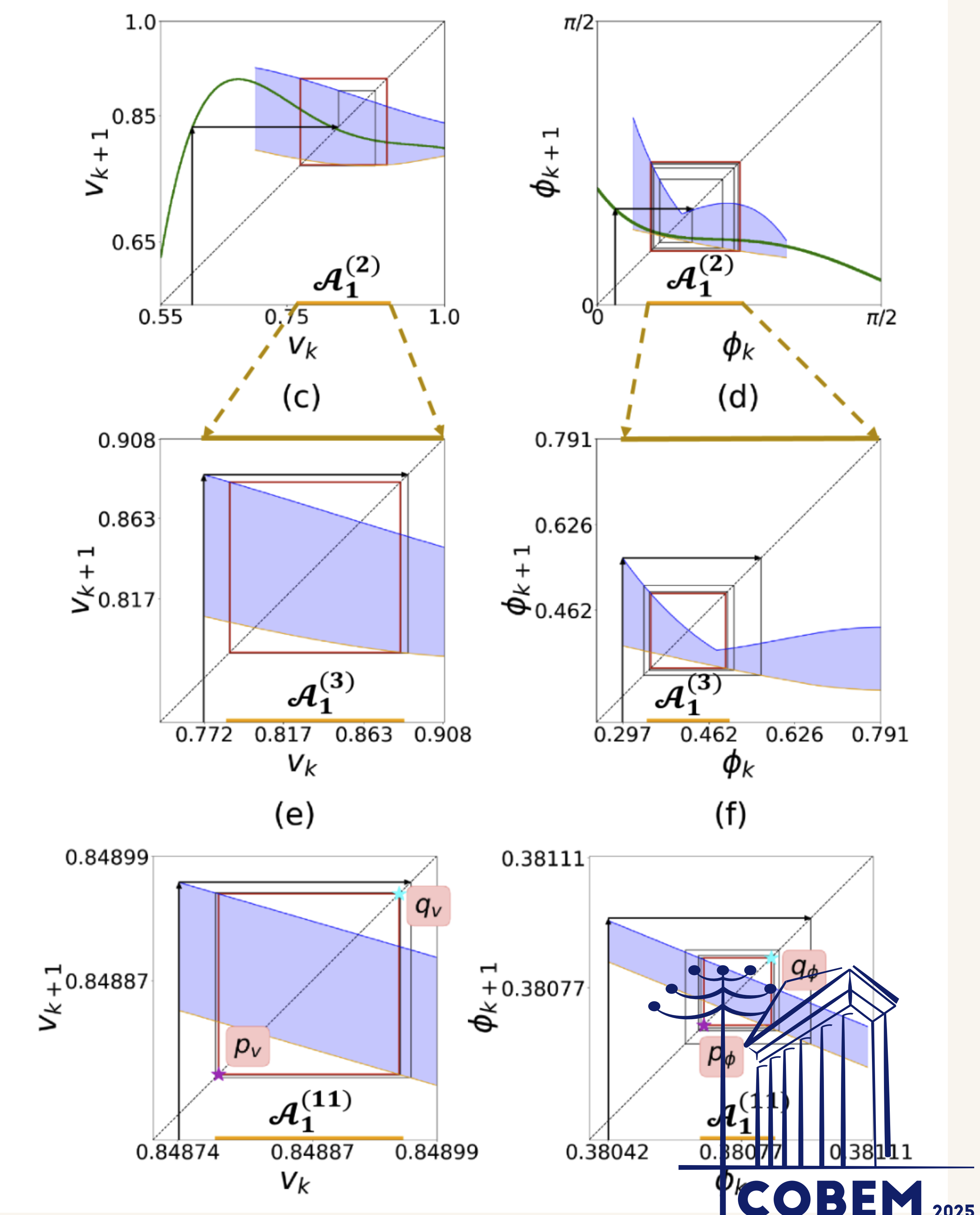


Computer-assisted method

Worst Case Scenario:

$$\eta_n^{(N)} = \begin{cases} \phi_{k+1} = \eta_{\max}^{(N)}(\phi_k), \quad \text{where} \;\; \eta_{\max}^{(N)} := \max_{v_k \in \mathcal{A}_n^{(N)}} \{g_n(v_k, \phi_k)\}, \\ \phi_{k+1} = \eta_{\min}^{(N)}(\phi_k), \quad \text{where} \;\; \eta_{\min}^{(N)} := \min_{v_k \in \mathcal{A}_n^{(N)}} \{g_n(v_k, \phi_k)\}. \end{cases}$$





CONCLUSIONS

Current State of the art

- Some substantial progress has been made in understanding of non-smooth system, including VI systems
- Almost anything is possible in such systems in terms of nonlinear behavior and bifurcations, including ghost bifurcations
- Global stability analysis can deal with low dimensional systems
- TET can be effective not only near the resonance, but also force should match the mass ratio for optimal performance
- Energy harvesting and Vibration Mitigation are not exactly two sides of one coin

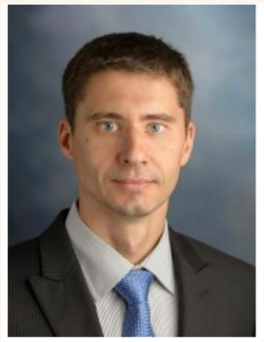
Challenges

- Large systems require high computational power tracking multiple impacts
- Automate the process of deriving maps
- Global stability analysis for MDOF systems
- Design systems with prescribed behavior (energy localization, energy transfer, etc)

ACKNOLEDGEMENT

Funding: NSF-CMMI 2009270 and EPSRC EP/V034391/1

Rachel Kuske, G. Tech.



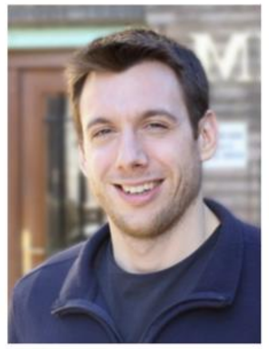
Igor Belykh, G. State

https://sites.google.com/view/vienergyharvest

Lanjing Bao, PhD student, Georgia State

Dr. Larissa Serdukova, now at U of Leicester, UK

Dr. Henrik Sykora, wow at Budapest University, Hungary



Dr. Dimitri Costa, now at Aberdeen University, UK

Dr. Christina Athanasouli, now at Williams College, USA

Dr. Rahul Kumar, now at IIT Bhubaneswar, India

COBEM 2025

QUESTIONS