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GRAVITY-DRIVEN FLOW OF GRAINS THROUGH PIPES: A ONE-DIMENSIONAL MODEL

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Abstract. *Grain flows through pipes are frequently found in industry, such as in pharmaceutical, chemical, petroleum, mining and food industries. In the case of size-constrained gravitational flows, density waves consisting of alternating high- and low-compactness regions may appear. This study investigates analytically the dynamics of density waves that appear in gravitational flows of fine grains through vertical and slightly inclined pipes. The length scales of density waves are determined using a one-dimensional model and a linear stability analysis. The analysis exhibits the presence of a long-wavelength instability, with the most unstable mode and a cut-off wavenumber whose values are in agreement with previously published results.*

Keywords: *Fine grains, gravitational flow, narrow pipes, instability, density waves*

1. INTRODUCTION

Gravitational grain flows in pipes are common in industry. Some examples are the transport of grains in the food industry, the transport of sand in civil constructions, and the transport of powders in the chemical and pharmaceutical industries. When the grains and the tube diameter are size-constrained, granular flow may give rise to instabilities. These instabilities consist of alternating high- and low-compactness regions (regions of high and low grain concentration, respectively), and are characterized by intermittency, oscillating patterns and even blockages (Raafat *et al.*, 1996; Aider *et al.*, 1999; Bertho *et al.*, 2002). Although this instability may appear under vacuum conditions (Savage, 1979; Wang *et al.*, 1997), in the case of fine grains these patterns are recognized as the result of the interaction between small-size falling grains and trapped air.

Lee (1994) investigated the density waves in granular flows through vertical tubes and hoppers using analytical techniques and numerical simulations. For the vertical tubes, the author found that kinetic waves exist and partially obtained a dispersion relation for the dynamic waves, which he did not solve. The numerical simulations were performed using molecular dynamics (MD), and the author found indications that the density waves are of kinetic nature. However, because air effects (pressure and drag) were absent in both the stability analysis and the numerical simulation, the results are not suitable in the case of fine grains in narrow pipes.

Raafat *et al.* (1996) studied the formation of density waves in pipes experimentally. The experiments were performed in a $1.3\,m$ long tube with an internal diameter D of $2.9\,mm$ using glass splinters and glass beads with mean grain diameter d of $0.09\,mm$ to $0.2\,mm$ and $0.2\,mm$, respectively. They observed density waves for moderate grain flow rate and when the ratio between the pipe and the grain diameter is $6 \leq D/d \leq 30$. Furthermore, they proposed that the friction between the grains and the forces between the trapped air and the grains are responsible for the density waves.

Aider *et al.* (1999) presented an experimental study of the granular flow patterns in vertical pipes. The experiments were performed in a tube similar to that of Raafat *et al.* (1996) using glass beads with mean diameter of $125\,\mu m$. The density variations were measured using a linear CCD (charge coupled device) camera with frequencies of up to $2\,kHz$. In addition, Aider *et al.* (1999) considered that the high-compactness plugs had compactness $c \approx 60\%$.

Bertho *et al.* (2002) presented experiments on density waves using an experimental set-up similar to that of Raafat *et al.* (1996) and Aider *et al.* (1999). The vertical tube ($D = 3\,mm$, $1.25\,m$ long) and the glass beads ($d = 125\,\mu m$ glass beads) were more or less the same as those of Aider *et al.* (1999), and a linear CCD camera was used. In addition, capacitance sensors were used to measure the compactness of grains at two different locations, and the pressure distribution was also measured. The experimental data showed that the characteristic length of the high-compactness regions of the density wave regime is in the order of $10\,mm$.

Recently, Franklin and Alvarez (2015) presented a linear stability analysis and experimental results for the vertical chute of grains in a narrow pipe. They found a dimensional dispersion relation to be solved numerically, and the analysis was limited to some small ranges of grains and pipes. The experiments were performed in a $1\,m$ long glass tube of $3\,mm$ internal diameter aligned vertically, and the grains consisted of glass beads of specific mass $\rho_s = 2500\,kg/m^3$ divided in two different populations: grains with diameter within $212\,\mu m \leq d \leq 300\,\mu m$ and within $106\,\mu m \leq d \leq 212\,\mu m$. Franklin and Alvarez (2015) reported the existence of granular plugs with length in the range $3 < \lambda/D < 11$, where λ is the plug length. Alvarez and Franklin (2017) presented an exhaustive set of experimental data and a dimensionless stability analysis that considers variations in local compactness. The analytical results are in agreement with the experimental data. The present paper reproduces many parts of Alvarez and Franklin (2017).

Numerical studies on intermittent granular flows in pipes have been carried out in recent years. Ellingsen *et al.* (2010) studied the gravitational flow of grains through a narrow pipe under vacuum conditions. They performed numerical simulations based on a one-dimensional model for the granular flow where the collisions were modeled using two coefficients of restitution, one among grains and the other between the grains and the pipe walls. A narrow pipe was assumed and periodic boundary conditions were employed. The numerical results showed that granular waves could form in the absence of air if the dissipation caused by the collisions among the grains was smaller than that between the grains and the walls. However, the proposed model cannot predict the wavelength of the density waves in the presence of interstitial gas.

The objective of the present study is to determine the wavelengths of density waves that appear when fine grains fall through vertical and slightly inclined pipes. This paper presents a one-dimensional flow model based on the work of Bertho *et al.* (2003) with the inclusion of closure equations for the friction terms, and also a linear stability analysis. The flow model is made dimensionless and the stability analysis takes into consideration the main mechanisms involved, namely the Janssen effect, the interaction between the grains and the air, and gravity, and the results are then compared to the experimental data.

The next sections describe the physics and the main equations of the one-dimensional model, the stability analysis of the granular flow, and the discussion of the main results. The conclusion section follows.

2. ONE-DIMENSIONAL TWO-PHASE MODEL

The analyzed problem consists of cohesionless fine grains falling from a hopper through a narrow tube. The ratio between the tube diameter and the mean grain diameter is within $6 \leq D/d \leq 30$, the humidity is within $35 < H < 75\%$, and the grain size and specific mass are such that the air effects are not negligible. The tube is in a vertical (or almost vertical) position, i.e., $-10^o \leq \theta \leq 10^o$, where θ is the pipe inclination with respect to the gravitational acceleration. Within this scope, density waves consisting of alternating high- and low- compactness regions are expected (Aider *et al.*, 1999; Bertho *et al.*, 2002). In the high-concentration regions, which are plugs of granular material, the compactness varies but is close to its maximum value, and grains in the plug periphery are in contact with the tube wall. Therefore, there is a redirection of forces within the plug and the Janssen effect is expected if the plugs are long enough (Duran, 1999; Cambau *et al.*, 2013). In the low-concentration regions, which are air bubbles with dispersed free-falling grains, the air pressure increases owing to the stresses caused by the neighboring plugs as well as the volume decrease caused by the free-falling grains. Figure 1 shows the layout of the gravitational granular flow.

A one-dimensional model is proposed for this problem. The model consists of an equation of motion for the grains in a compact plug (or a compact regime), an air pressure equation, and a mass conservation equation of the grains. These equations, displayed below, are used in the stability analysis in Section 3. The stability analysis is performed to determine the length scale of the granular plugs.

2.1 Mass conservation of the grains

The mass transport equation of the grains is given by Eq. 1.

$$\frac{\partial c}{\partial t} + c\frac{\partial v_s}{\partial z} + v_s\frac{\partial c}{\partial z} = 0 \tag{1}$$

where c is the compactness of granular plugs (granular volume fraction), v_s is the local grain velocity, z is the vertical coordinate, and t is the time. Normalizing Eq. 1 by the characteristic length $L_c = D$, time $t_c = \sqrt{D/g}$, and velocity $v_c = \sqrt{gD}$, we obtain Eq. 2:

$$\frac{\partial c}{\partial t^*} + c\frac{\partial v_s^*}{\partial z^*} + v_s^*\frac{\partial c}{\partial z^*} = 0 \tag{2}$$

where $z^* = z/D$, $t^* = t/\sqrt{D/g}$, ${v_s}^* = v_s/\sqrt{gD}$.

2.2 Granular motion

The equation of motion for the grains in compact regime is given by a balance between the grains acceleration, the friction between the grains and the tube wall, the forces due to air pressure and granular tension distribution, and the weight. This balance is given by Eq. 3,

$$\rho_s \left(\frac{\partial c v_s}{\partial t} + \frac{\partial c v_s^2}{\partial z} \right) = \rho_s \, cg \, \cos\theta - \frac{\partial P}{\partial z} - \frac{\partial \sigma_{zz}}{\partial z} - \frac{4}{D}\sigma_{zr} \tag{3}$$

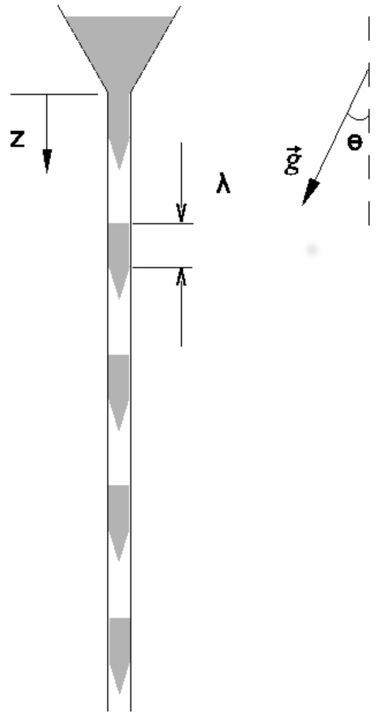


Figure 1. Layout of the gravitational granular flow through a narrow pipe. z is the vertical coordinate, λ is the length of the granular plugs, and θ is the pipe inclination with respect to the gravitational acceleration $\vec{g}$. (Figure extracted from Alvarez and Franklin (2017)).

where ρ_s is the specific mass of each grain, g is the gravitational acceleration, P is the air pressure, σ_{zz} is the vertical stress operating on the grains, and σ_{zr} is the stress between the tube wall and the grains. We use here the closure of σ_{zz} and σ_{zr} proposed by Franklin and Alvarez (2015). The first one is to take into account the redirection of forces through a constant coefficient (dimensionless) (Duran, 1999) κ: $\sigma_{zr} = \mu_s \kappa \sigma_{zz}$, where $\mu_s \approx \tan(32^o)$ is the friction coefficient between the grains, and the grains and the pipe walls. The second is to model σ_{zr} as a function of the square of the grains velocity: $\sigma_{zr} = 1/2\rho_s \mu_s {v_s}^2$. The third is to consider that capillary forces can be modeled as a multiplicative constant $b \geq 1$ on the friction term. This is a simple way to take into account capillary forces, that act in the same direction as the friction forces. However, for this study, we fixed $b = 1$ and did not change it. The resulting equation is

$$\frac{\partial c v_s}{\partial t} + \frac{\partial c v_s^2}{\partial z} = c \, g \, \cos\theta - \frac{1}{\rho_s}\frac{\partial P}{\partial z} - \frac{v_s}{\kappa}\frac{\partial v_s}{\partial z} - \frac{2}{D}\mu_s v_s^2 \tag{4}$$

Normalizing Eq. 4 by the characteristic length L_c, time t_c, velocity v_c and pressure $P_c = \rho_s g D$, we obtain Eq. 5

$$c\frac{\partial v_s^*}{\partial t^*} + v_s^* \left(\frac{\partial c}{\partial t^*} + v_s^* \frac{\partial c}{\partial z^*} + c\frac{\partial v_s^*}{\partial z^*} \right) + c v_s^* \frac{\partial v_s^*}{\partial z^*} = c \, \cos\theta - \frac{\partial P^*}{\partial z^*} - \frac{v_s^*}{\kappa}\frac{\partial v_s^*}{\partial z^*} - 2\mu_s (v_s^*)^2 \tag{5}$$

In Eq. 5 we identify the normalized mass conservation equation (Eq. 2); therefore, with an additional simplification, we obtain Eq. 6.

$$c\frac{\partial v_s^*}{\partial t^*} + c v_s^* \frac{\partial v_s^*}{\partial z^*} - c \, \cos\theta + \frac{\partial P^*}{\partial z^*} + \frac{v_s^*}{\kappa}\frac{\partial v_s^*}{\partial z^*} + 2\mu_s (v_s^*)^2 = 0 \tag{6}$$

where $P^* = P/(\rho_s g D)$

2.3 Air pressure

For the air pressure, an equation based on the work of Bertho *et al.* (2003) is used. Bertho *et al.* (2003) combined the mass conservation equations for the air and grains, the isentropic relation for the air, and Darcy's equation relating the air flow through packed grains to the pressure gradient to obtain Eq. 7

$$\frac{\partial P}{\partial t} + v_s \frac{\partial P}{\partial z} + \frac{\gamma P}{(1-c)} \frac{\partial v_s}{\partial z} - B \frac{\partial^2 P}{\partial z^2} = 0 \tag{7}$$

where γ is the ratio of specific heats (1.4 for air) and B is a coefficient given by

$$B = \frac{\gamma P \left(1-c\right)^2 d^2}{\mu_a 180 c^2} \tag{8}$$

where μ_a is the dynamic viscosity of air. In Eq. 8, B was obtained by estimating the permeability of grains using the Carman–Kozeny equation. Normalizing Eq. 7 by the characteristic length L_c, time t_c, velocity v_c and pressure P_c, we obtain Eq. 9

$$\frac{\partial P^*}{\partial t^*} = -v_s^* \frac{\partial P^*}{\partial z^*} - \frac{\gamma P^*}{(1-c)} \frac{\partial v_s^*}{\partial z^*} + B^* \frac{\partial^2 P^*}{\partial z^{*2}} \tag{9}$$

where $B^* = B g^{-1/2} D^{-3/2}$ is a dimensionless coefficient.

3. STABILITY ANALYSIS

A linear stability analysis is presented based on Eqs. 2, 6 and 9, which are solved for P^*, c and v_s^*. The main objective is to find the typical length for the high-density regions of the granular flow. The initial state, considered as the basic state, is a steady, dense uniform flow of grains known as compact regime (Aider *et al.*, 1999). This state is then perturbed and we investigate if a preferential mode exists, i.e., we investigate if an initial compact regime will be fractured in granular plugs with a preferential wavelength. Thus, the analysis considers a basic state in which the pressure is equal to the characteristic pressure, P_c, the grain velocity is equal to the characteristic velocity v_c, and the compactness is equal to an average dense compactness c_0 (where, in compact regime $c_0 \approx 0.55$). The pressure, grain velocity and compactness are then the sum of the basic state, of $O(1)$, and the perturbation, of $O(\epsilon)$, $\epsilon \ll 1$. In dimensionless form:

$$P^* = 1 + \tilde{P}, \qquad v_s^* = 1 + \tilde{v}_s, \qquad c = c_0 + \tilde{c} \tag{10}$$

where $\tilde{P} \ll 1$, $\tilde{v}_s \ll 1$ and $\tilde{c} \ll 1$ are respectively the pressure, velocity and compactness perturbations (dimensionless).

By inserting the pressure, the velocity and the compactness from Eq. 10 in Eqs. 2, 6 and 9, and keeping only the terms of $O(\epsilon)$, we obtain

$$\frac{\partial \tilde{c}}{\partial t^*} + c_0 \frac{\partial \tilde{v}_s}{\partial z^*} + \frac{\partial \tilde{c}}{\partial z^*} = 0 \tag{11}$$

$$c_0 \frac{\partial \tilde{v}_s}{\partial t^*} + c_0 \frac{\partial \tilde{v}_s}{\partial z^*} - \tilde{c} \cos\theta + \frac{\partial \tilde{P}}{\partial z^*} + \frac{1}{\kappa} \frac{\partial \tilde{v}_s}{\partial z^*} + 4 \mu_s \tilde{v}_s = 0 \tag{12}$$

$$\frac{\partial \tilde{P}}{\partial t^*} = -\frac{\partial \tilde{P}}{\partial z^*} - \frac{\gamma}{(1-c_0)} \frac{\partial \tilde{v}_s}{\partial z^*} + B_1^* \frac{\partial^2 \tilde{P}}{\partial z^{*2}} \tag{13}$$

where B_1^* is a constant obtained by replacing P by P_c in B^*. Equations 11, 12 and 13 form a linear system with constant coefficients; therefore, the solutions can be found by considering the following normal modes:

$$\begin{aligned} \tilde{c} &= \hat{c}\, e^{i(k^* z^* - \omega^* t^*)} + c.c. \\ \tilde{v}_s &= \hat{v}_s\, e^{i(k^* z^* - \omega^* t^*)} + c.c. \\ \tilde{P} &= \hat{P}\, e^{i(k^* z^* - \omega^* t^*)} + c.c. \end{aligned} \tag{14}$$

where $k^* = kD = 2\pi D/\lambda \in \mathbb{R}$ is the dimensionless wavenumber in the z^* direction, λ is the wavelength in the z^* direction, $\hat{c} \in \mathbb{C}$, $\hat{v} \in \mathbb{C}$ and $\hat{P} \in \mathbb{C}$ are the dimensionless amplitudes, and $c.c.$ stands for the complex conjugate. Let $\omega^* \in \mathbb{C}$, $\omega^* = \omega_r^* + i\omega_i^*$, where $\omega_r^* = \omega_r/(kv_c) \in \mathbb{R}$ is the dimensionless angular frequency and $\omega_i^* = \omega_i t_c \in \mathbb{R}$ is the dimensionless growth rate. By inserting the normal modes in Eqs. 11, 12 and 13, we obtain Eq. 15.

$$\begin{bmatrix} ik^* - i\omega^* & ik^* c_0 & 0 \\ -\cos\theta & -i\omega^* c_0 + ik^* c_0 + \kappa^{-1} ik^* + 4\mu_s & ik^* \\ 0 & \frac{\gamma}{1-c_0} ik^* & -i\omega^* + ik^* + B_1^* k^{*2} \end{bmatrix} \begin{bmatrix} \hat{c} \\ \hat{v_s} \\ \hat{P} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

The existence of non-trivial solutions for this system requires its determinant to be zero. This results in

$$\begin{gathered} ic_0\,\omega^{*3} + \omega^{*2}\left[k^*(-3ic_0 - i\kappa^{-1}) - k^{*2} c_0\, B_1^* - 4\mu_s\right] + \\ +\omega^*[k^{*3}(2c_0\, B_1^* + \kappa^{-1} B_1^*) + ik^{*2}(2\kappa^{-1} - \tfrac{\gamma}{1-c_0} + 3c_0 - 4\mu_s B_1^*) + k^*(c_0\,\cos\theta + 8\mu_s)] + \\ +ik^{*3}\left(-c_0 - \kappa^{-1} + 4\mu_s B_1^* + c_0\,\cos\theta\, B_1^* + \tfrac{\gamma}{1-c_0}\right) + k^{*2}\left(-4\mu_s - c_0\,\cos\theta\right) + B_1^* k^{*4}\left(-\kappa^{-1} - c_0\right) = 0 \end{gathered} \tag{16}$$

Equation 16 is solved to find $\omega^*(k^*)$. In order to solve it, constant b was assumed to be equal to 1. Constant B_1^* was computed using the characteristic values, i.e., $D = 3\,mm$, $d = d_{50} = 0.225\,mm$, and we assumed that $\mu_s = \tan(32^o)$, $\kappa \approx 0.5$, and $c_0 \approx 0.55$ (Duran, 1999; Cambau *et al.*, 2013). Additionally, we assumed $\theta \leq 5^o$. The imaginary part of $\omega^*(k^*)$, $\omega_i^*(k^*)$, was investigated in order to obtain the typical length of granular plugs.

Figure 2 shows the dimensionless growth rate ω_i^* as a function of dimensionless wavenumber k^*. The continuous line corresponds to one root of Eq. 16, the dashed-dot line and the dashed line to the others. Figure 2a, for the broad range of wavenumbers, shows that small wavelengths are stable. Figure 2b illustrates the $0 \lesssim kD \lesssim 0.8$ region. The figure shows there is a solution, given by the continuous line, which corresponds to a long-wavelength instability, with a preferential mode in $k^* \approx 0.2$ and a cut-off wavelength of $k^* \approx 0.6$. This corresponds to wavelengths in the order of $10D$.

4. RESULTS AND DISCUSSION

These experiments and those of Franklin and Alvarez (2015) showed that the plug sizes are $3 < \lambda/D < 11$, which is in perfect agreement with the proposed model. However, as only one tube diameter was employed, we then compare these results with the previously published results.

In a series of papers, Raafat *et al.* (1996), Aider *et al.* (1999), and Bertho *et al.* (2002) presented experiments of granular flows through a tube. In particular, with regard to the characteristics of density waves, Raafat *et al.* (1996) reported the size of plugs was $\lambda/D \approx 10$ and that it was approximately independent of the flow rate. Bertho *et al.* (2002) also reported that the size of plugs was $\lambda/D \approx 10$. In addition, they showed the length of air bubbles is $\lambda_{bubble}/D \approx 10$. These measurements are in agreement with the lengths predicted by the proposed model.

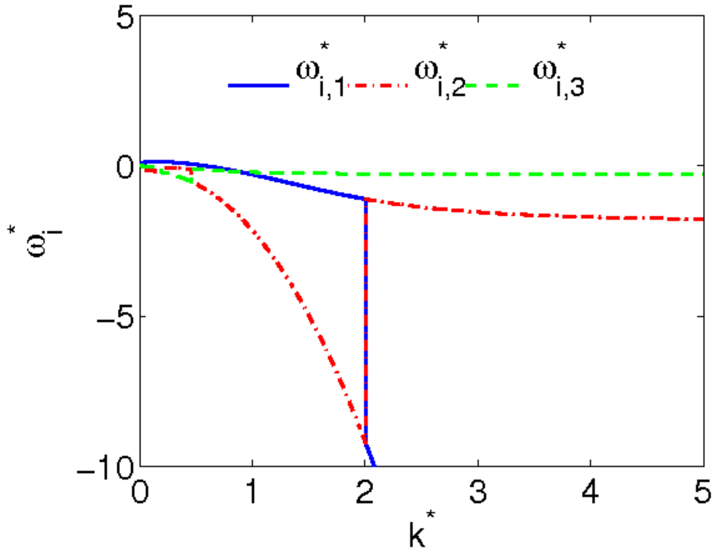
The results of the model are in agreement with the results reported by Franklin and Alvarez (2015), even though in the present study the compactness c was assumed as a variable, different from that work, where the authors fixed c as constant. In addition, through θ it is possible to consider small deviations of the tube with respect to the vertical alignment. This allows to take into consideration small angle variations that may have occurred in the cited works.

As far as we know, Lee (1994) performed the only stability analysis previous to our work. In his analysis, Lee neglected air effects (pressure and drag); therefore, the analysis was not able to find the correct length scale of plugs. Different from Lee (1994), we considered air effects, and the present stability analysis predicts a wavelength that agrees with typical lengths observed in different experiments.

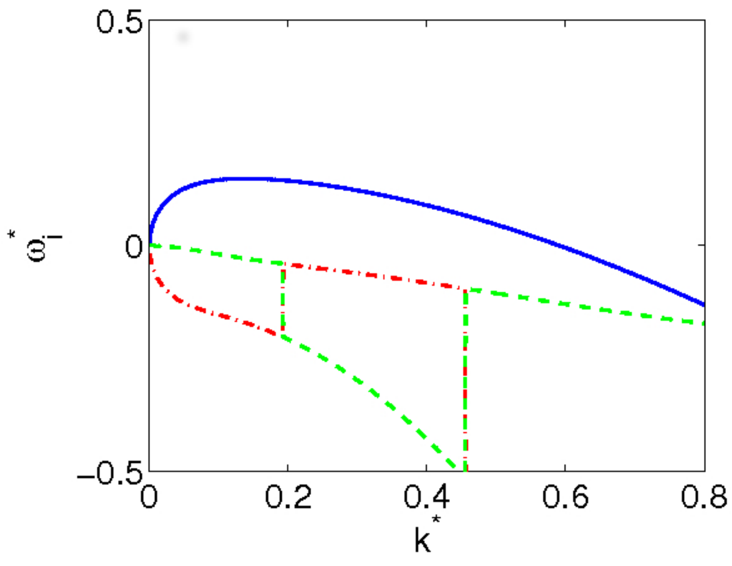
The final observation concerns the lowest plug in the gravitational dense flow. Bertho *et al.* (2002) reported that at the lower portion of the tube (tube exit) a different plug is formed. The length of this plug varies with the flow rate. For $\dot{m}$ from $1.75\,g/s$ to $3.9\,g/s$, they found that the length of the bottom plug varies from $\lambda/D \approx 30$ to $\lambda/D \approx 200$. This plug is subject to exit boundary conditions; therefore, its length is not correctly predicted by the present analysis.

5. CONCLUSIONS

This paper focused on the density waves that appear when fine grains fall through a narrow tube. Its main objective was to analytically determine the wavelengths of density waves that appear when fine grains fall through vertical and



(a)



(b)

Figure 2. Dimensionless growth rate ω_i^* as a function of dimensionless wavenumber k^*. The continuous line corresponds to one root of Eq. 16, the dashed-dot line and the dashed line to the others. (Figure extracted from Alvarez and Franklin (2017)).

slightly inclined pipes. This study presented a stability analysis based on equations proposed by Bertho *et al.* (2003), with small modifications and in dimensionless form. In our analysis, the basic state is a compact regime, and we investigated if it would be fractured in granular plugs with a preferential wavelength. The stability analysis predicts a wavelength in the order of $10D$ for the high-density regions. This predicted length scale is in good agreement with previously published results.

6. ACKNOWLEDGEMENTS

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