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NUMERICAL SIMULATION OF TWO-PHASE SLUG FLOW FROM HORIZONTAL TO UPWARD INCLINED PIPE USING A HYBRID CODE BASED ON SLUG TRACKING AND SLUG CAPTURING METHODOLOGIES

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Abstract. *This work presents a new transient hybrid model to simulate the gas-liquid slug flow pattern in pipes. A slug tracking methodology was integrated to a two-fluid slug capturing model and, as a result, the new model will be able to simulate the initiation of the slug flow from the stratified pattern and track the unit cells (structures composed by a slug and an elongated bubble) with a low computational effort. A simulation was carried out for a* $40\ [m]$ *pipe, with a change of direction from horizontal to a* 5^o *upward inclined pipe. The results and computational cost of this new hybrid code were compared to those from the slug capturing. A reduction of more than* 50% *in the computational effort along with similar results for bubble length, slug length, slug frequency and pressure gradient was achieved.*

Keywords: *Slug Flow, Slug Capturing, Slug Tracking, Two-phase flow*

1. INTRODUCTION

The gas-liquid slug flow is characterized by an unevenly distribution of the phases into two different structures: an elongated gas bubble and a liquid slug, as represented in Fig. 1. Slug flow is perhaps the most common flow pattern in oil and gas production, and appears in the nuclear industry as well. Different methodologies to simulate slug flows are found in the literature.

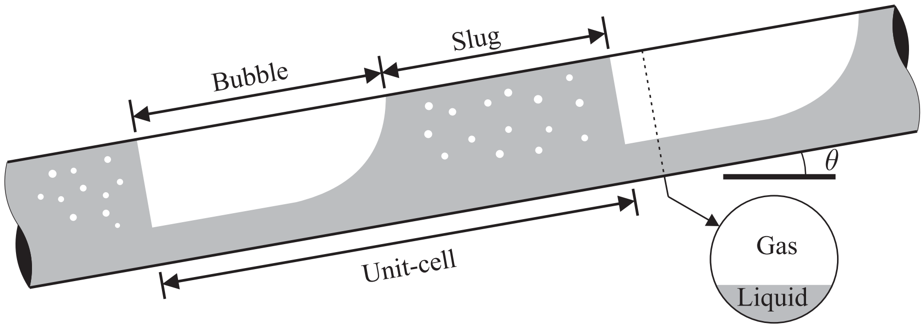


Figure 1: Physical model of the gas-liquid slug flow pattern.

The first models assumed a repetition of the structures, liquid slug and elongated bubble, in time and space (Wallis, 1969; Dukler and Hubbard, 1975; Fernandes, Semiat and Dukler, 1983; Taitel and Barnea, 1990). The slug may have dispersed bubbles, and the bubble flows over a film of liquid. Analytical and experimental correlations were used to obtain the main parameters. These mechanistic models are still in use, due to the low computational effort. Transient models have also been developed. Some well-known models are the two-fluid (Ishii, 1975), drift-flux (Zuber and Findlay, 1965; Wallis, 1969), and slug tracking (Taitel and Barnea, 1993, 1998; Franklin and Rosa, 2004) models.

Recently, a simplified two-fluid Lagrangian model was presented by Renault (2007). The model is capable of simulating the transition from stratified to slug flow. The equations for the gas in the elongated bubble and liquid in the slug are solved by means of a finite-difference scheme, whereas the equations for the liquid in the film, similar to the shallow water equations, is solved by means of an analytical solution for the Riemann problem. Based on this model, Conte (2014) and Conte *et al.* (2014) developed a code to simulate slug initiation. The results from simulations were compared to experimental data, and a good agreement was found. The disadvantage of the model is the relatively high computational cost.

The Rodrigues (2009) slug tracking model applies the continuity equation to the two structures of the slug flow, and the momentum equation to the slug, in order to derive two differential equations, where the most important variables are the pressure in the elongated bubble and the liquid velocity in the slug. The model is solved by a finite-difference scheme, and the other variables (film velocity, slug length and bubble length) are solved through auxiliary relationships. The boundaries are free to move and follow the unit cells. The model has a low computational cost, but information about the unit cells that enter the pipe are required.

The focus of this article are the slug capturing and the slug tracking models and its objective is to use an hybrid model to simulate slug flows, applying a methodology that simultaneously uses both models. The two-fluid model will be used to simulate the slug initiation and the change of direction whereas the slug tracking model will simulate the slug propagation. The new hybrid code will simulate the flow in a 40 [m] long, 0.026 [m] diameter pipe, with a change of direction from horizontal to 5^o upward inclined, and the results will be compared to those obtained from the slug capturing code.

2. METHODOLOGY

The first two parts of this section briefly present the slug capturing (Conte, 2014; Renault, 2007) and the slug tracking (Rodrigues, 2009) models, summarizing the mathematical and numerical modelling. The former model will be used to simulate the slug initiation and the change of direction, whereas the latter, due to its low computational cost, will be used to simulate the slug flow in straight sections. In the third part of the section, the methodology that simultaneously uses both models will be explained in details.

2.1 Slug capturing model

A simplified two-fluid model uses the continuity and momentum equations for the gas and the liquid, where such phases are equated independently. The goal is to model the transition from stratified to slug flow. The liquid is incompressible and the gas is ideal. The temperature is assumed to be constant with no mass change. The following equations express mass and momentum conservation for each phase:

$$\frac{\partial}{\partial t}(R_L) + \frac{\partial}{\partial x}\left(R_L U_L\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_G R_G) + \frac{\partial}{\partial x}\left(\rho_G R_G U_G\right) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(R_L U_L) + \frac{\partial}{\partial x}\left(R_L U_L^2 + \frac{1}{2}\kappa R_L^2\right) = \frac{R_L}{\rho_L} F(R_L, U_L, U_G) \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_G R_G U_G) + \frac{\partial}{\partial x}\left(\rho_G R_G U_G^2\right) = \left(-\frac{\tau_G S_G}{A} - \frac{\tau_I S_I}{A} - \rho_G g R_G \sin\theta - R_G \frac{\partial p}{\partial x}\right) \tag{4}$$

where the symbol ρ is the density, t is the time, g is the gravity, R is the phase volume fraction, U is the velocity, τ is the shear stress, p is the pressure, A is the cross-sectional area and S is the wetted perimeter. The subscript L denotes the liquid phase, G the gas phase and I the gas-liquid interface. Equations 1 and 2 represent the continuity, and Eqs. (3) and (4) represent the momentum. The volumetric forces are expressed by the following equation:

$$F = -\frac{\tau_L S_L}{A_L} + \frac{\tau_G S_G}{A_G} + \tau_I S_I \left(\frac{1}{A_L} + \frac{1}{A_G}\right) - (\rho_L - \rho_G) g \sin\theta \tag{5}$$

The Lagrangian solution uses a grid which is split into cells (see Fig. 2), and each cell can be either a stratified section or a slug. An elongated bubble may be divided into several sections. The boundaries are free to move, but each section is restricted to a maximum length, L_{max}, in order to maintain mesh refinement.

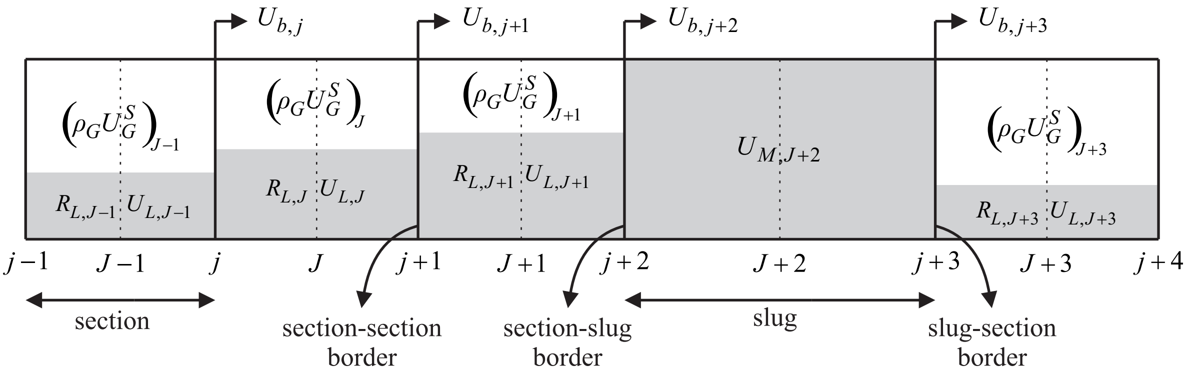


Figure 2: Computational grid for the slug capturing model.

A computational section J carries the gas flux, $\left(\rho_G U_G^S\right)_J$, the liquid velocity, U_L, and the liquid hold-up, R_L. The slug, e.g. cell $J+2$, has the mixture velocity $U_{M,J+2}$. Each border j has the pressure, p_j, and the border velocity, $U_{b,j}$. The slug is modelled as non-aerated.

The first step of the solution applies the gas equations to every section and the liquid equations to each slug. The gas phase within the sections and slug domain is solved using a implicit finite-difference scheme. A system of equation $a_J X_J = b_J X_{J+1} + c_J X_{J-1} + d_J$ is obtained, where X_J could be either the gas flux or the slug velocity. This part of the solution provides the simultaneous computation of pressure, gas flux and slug velocity.

The next step is the film solution. It is divided in two parts. In the first part, only the influence of the volumetric forces is taken into consideration to calculate the intermediate liquid velocity $U_{L,J}^{n+1/2}$, meaning that the convection term of eq. 3 is neglected. The next step, on the other hand, neglects the volumetric forces of eq. 3. The associated equation is modeled similarly as the shallow water equations and it uses the solution of the Riemann's problem. It provides the updated liquid hold-up and liquid velocity for each section. Additionally, the borders velocity U_b are defined in order to follow the shock waves. The bubble front is calculated as proposed by Dukler and Hubbard (1975), including the wake effect (pressure loss due to the recirculation of liquid from the film to the slug) as used by Rodrigues (2009).

The last step of the solution consists of the list management. Since the model is Lagrangian, section are free to grow. A section J may be merged to an adjacent cell if it does not satisfy the CFL condition or be split in two sections if $L_J \geq L_{max}$. Furthermore, a section becomes a slug if $R_{L,J} \geq 0.98$, and the volume required to reach $R_{L,J} = 1$ is subtracted from the neighboring sections. As boundary conditions, the model needs the pressure in the last cell, and the superficial velocities in the first cell.

2.2 Slug tracking model

The slug tracking model uses the continuity equation for the gas and the liquid in the elongated bubble and in the liquid slug, and the momentum equation for the liquid in the slug. In this article, the slug is considered non-aerated. It is possible to obtain the average velocity of the liquid for each slug, $U_{LS,i}$, the pressure in the elongated bubble, $p_{GB,i}$, the velocity of the liquid, $U_{LB,i}$, the translational velocity of the bubble, $U_{T,i}$, and the positions of the boundaries, x_i and y_i, as indicated in Fig. 3.

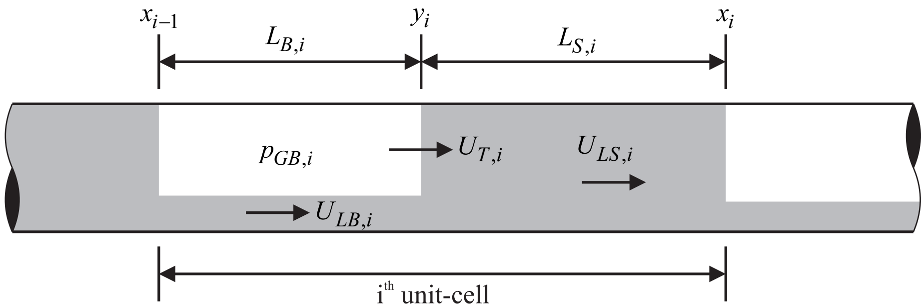


Figure 3: Unit cell for the slug tracking model

The following equations express the mass balance in the unit cell and the momentum balance in the slug:

$$U_{LS,i-1} - U_{LS,i} = \frac{dp_{GB,i}}{dt}\left[L_{B,i}\frac{(1-R_{LB,i})}{p_{GB,i}}\right] \tag{6}$$

$$
\begin{aligned}
p_{GB,i} - p_{GB,i+1} = & \rho_L L_{S,i} \frac{dU_{LS,i}}{dt} + 2C_{LS,i}\rho_L \frac{L_{S,i}}{D} U_{LS,i}^2 + 2C_{LB,i+1}\rho_L s_{LB,i+1} \frac{L_{B,i+1}}{D} U_{LB,i}^2 \\
& + (L_{S,i} + R_{LB,i+1} L_{B,i+1})\rho_L g \sin\theta
\end{aligned}
\tag{7}
$$

A semi-implicit difference scheme is applied to the eqs. 6 and 7. Pressure in the last bubble and mixture velocity in the slug are calculated by solving the system of equations. The bubble front is also calculated as proposed by Dukler and Hubbard (1975) and Rodrigues (2009), and the slug front is calculated by means of a mass balance for the gas phase. The liquid hold-up in the film is kept constant. This model needs the information from the cell that is being inserted (slug length, bubble length, pressure and slug velocity). The boundary condition is the pressure in the last bubble.

2.3 Hybrid methodology

As previously mentioned, the purpose of this work is to use an hybrid code to simulate slug flows (see Fig. 4). The slug capturing model presented in subsection 2.1 will be used to simulate the transition from stratified to slug flow, and the slug tracking model presented in subsection 2.2 will be used to simulate the propagation of the slugs. At the entrance of the pipe, the slug capturing model simulates the initiation of the flow and, after six slugs are generated, the slug tracking model is invoked. In a distance of $40D$ (where D is the pipe diameter) before the elbow, the slug capturing code is invoked again to simulate the flow behaviour during the change of direction, and after six slugs the slug tracking code is used. A detailed explanation on those steps is given later.

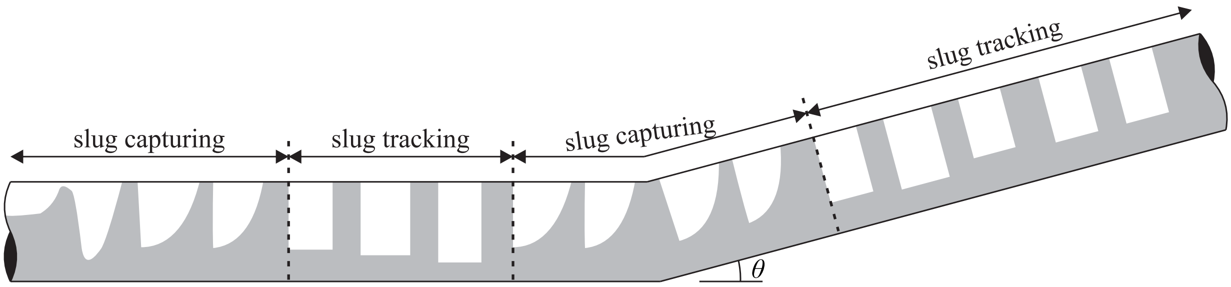


Figure 4: Hybrid methodology to simulate the slug flow in pipes.

The models interface in such a way that the information that one model needs as a boundary condition will be provided by the other model. Figure 7 shows how the models are linked.

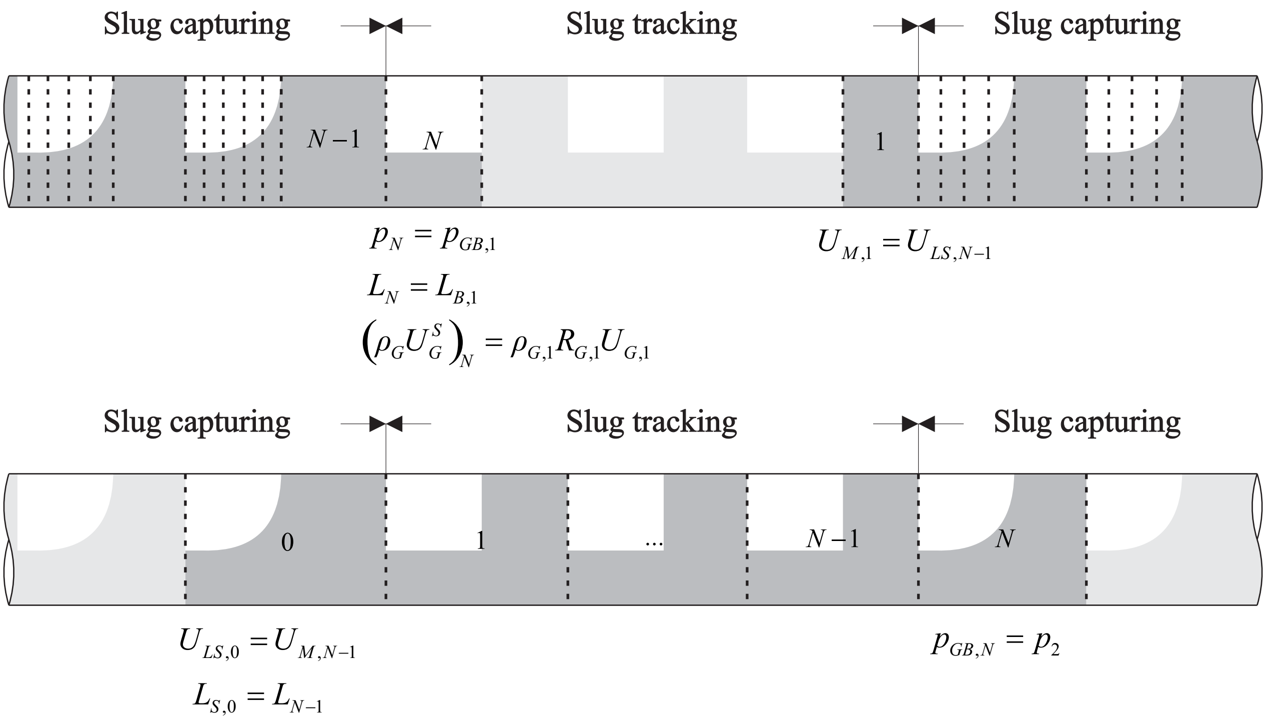


Figure 5: Coupling of the models.

The interface between the models will always be placed in a border between the front of a slug and the back of a bubble, meaning that the borders can move freely. Figure 6 shows in details how the boundary conditions are provided for each model.

Procedures to get the information at the interfaces

a. For the slug capturing model (see Fig. 5a):
- The pressure in the last cell will be the pressure in the first bubble of the slug tracking model, $p_N = p_{GB,1}$;
- The length of the last cell will be the length of the first bubble of the slug tracking model, $L_N = L_{B,1}$;
- The gas flux is also computed from the information supplied by the slug tracking model, $\left(\rho_G U_G^S\right) = \rho_{G,1} R_{G,1} U_{G,1}$;
- The mixture velocity at the first slug is taken from the last cell of the slug tracking model, $U_{M,1} = U_{LS,N-1}$.

b. For the slug tracking model (see Fig. 5b):
- Pressure in the cell that is being inserted is taken from the last cell of the slug capturing model, $p_{GB,0} = p_N$;
- Slug velocity, slug and bubble length of the cell that is being inserted is also taken from the last cell of the slug capturing model, $U_{LS,0} = U_{M,N-1}$ and $L_{S,0} = L_{N-1}$;
- The boundary condition at the exit (pressure) is taken from the second cell of the slug capturing model, $p_{GB,N} = p_2$.

Figure 6: Boundary condition treatment at the interfaces between models

As said before, the interfaces between the models move freely. When a slug passes from the slug capturing model to the slug tracking one, the entire unit cell is transitioned as indicated in Fig. 7a. The cell is eliminated from the slug capturing and transferred to the new unit cell of the slug tracking. The average properties (slug velocity, void fraction, bubble and slug lengths) must remain the same, in order to satisfy mass conservation for the two phases

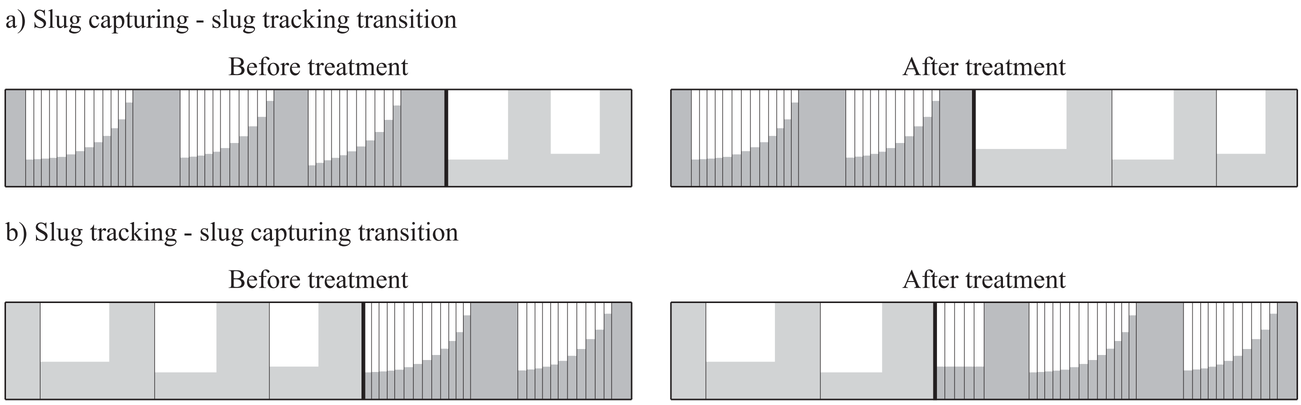


Figure 7: Computational transition of cells between the models.

When a front of bubble is $40D$ distant from the elbow, the entire unit cell changes from the slug tracking to the slug capturing model, as indicated in Fig. 7b. Since the slug tracking methodology does not compute the shape of the elongated bubble, it is assumed that the film has a constant level of liquid. The entire unit cell is erased from the slug tracking and transferred to the slug capturing model; the sections are created with equal sizes dx.

Figure 8 illustrates the start of the simulation and the main steps until the flow stabilizes. As a initial condition, the flow is assumed as stratified in $t = 0$ and only the slug capturing code works. After the generation of six slugs, in $t = t_2$, the slug tracking code is called. When a bubble front is close to the elbow (by a distance smaller than $40D$), in $t = t_3$, it is transferred from the slug capturing to the slug tracking (see Fig. 7b). For $t = t_4$ after five slugs have passed throw the elbow, the slug tracking code is used. In $t = t_5$ the slugs reached the pipe outlet and the slug capturing code no longer simulates the flow at the outlet of the pipe.

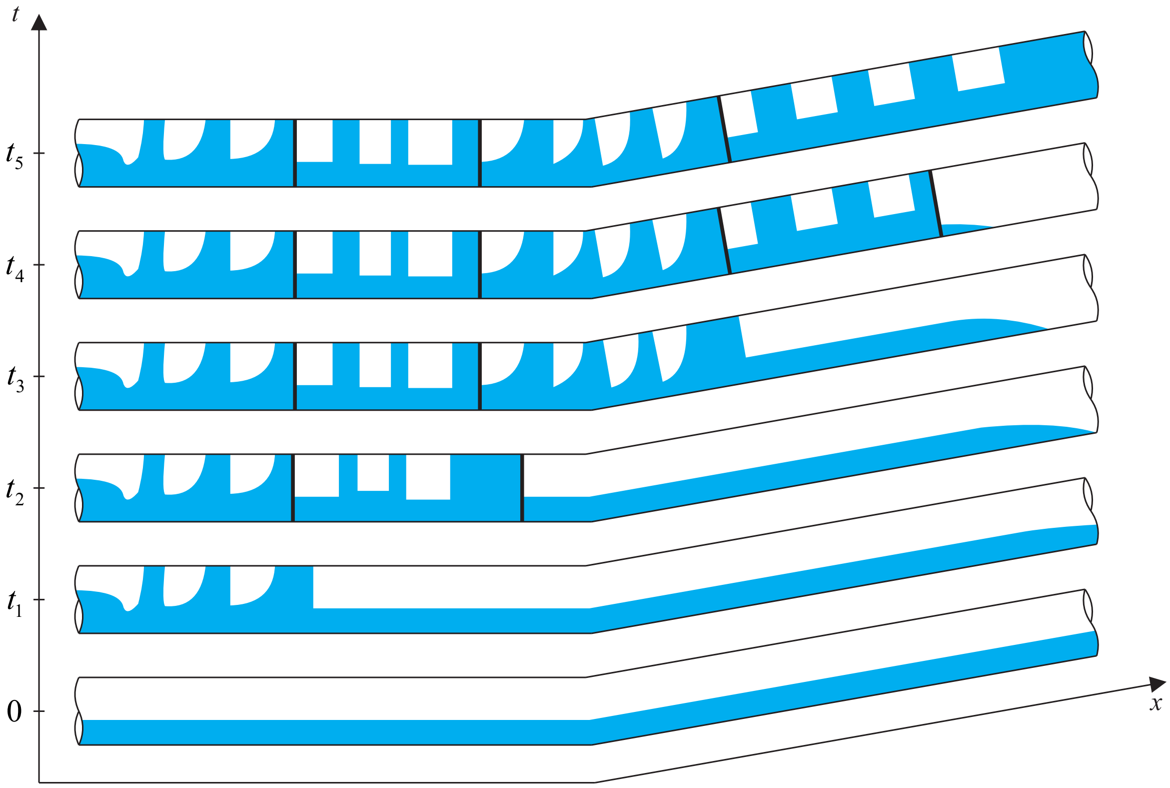


Figure 8: Stratified flow as initial condition.

As a boundary condition, the pressure at the outlet and the superficial velocities at the inlet are necessary. In general, the pipe may be divided into several sections, where each section is associated to one of the two models. For example, in $t = t_5$ (see Fig. 8) the pipe has four sections (or four models). In order to solve the problem for the entire pipe, it is necessary to solve each one of the sections for each time step.

Each model must be solved individually. The solution starts at the last section, the one adjacent to the outlet of the pipe. The model linked to that section must be solved and the boundary conditions updated. Then the next section can be solved and the boundary conditions linked to the section are updated once again. The process is the same for the remaining sections, solving each model linked to the respective section until the first one, adjacent to the inlet of the pipe. The next important step is the management of the interfaces between models, when some unit cell may be transferred from one model to the other (illustrated in Fig. 7). The last step is the management of the list of sections (or the models linked to the sections). Some models may be inserted or deleted whenever necessary, as represented in Fig. 8. Figure 9 summarizes the major steps to solve the problem with the hybrid methodology.

Hybrid methodology: major steps

1. Each one of the models is solved, starting with the last one (outlet) and ending at the first one (inlet);
2. Management of the interface positions;
3. Management of the list of models.

Figure 9: Procedures to solve the problem using the hybrid model

For large pipe extensions, two-fluid model-based codes would take a long time to perform simulations. The new approach presented in this work aims at applying the slug tracking model to most of the pipe extension. In the next section, some results of the hybrid model will be presented.

3. RESULTS

Two simulations were carried out: one using the hybrid code and another using the slug capturing code. It was selected the velocity pair $U_G^S = 0.3$ $[m/s]$ and $U_L^S = 0.7$ $[m/s]$ in a pipe of diameter $D = 0.026$ $[m]$ and a total length of $L = 40$ $[m]$, divided into two sections of 20 $[m]$ each. The first section is horizontal and the second one has an upward inclination of 5^o. The time step is $dt = 0.005$ $[s]$ and the initial mesh size is $dx = 0.01$ $[m]$. In this work, $L_{max} = 2dx$. As a initial condition, stratified flow with liquid hold-up $R_L = 0.05$ was assumed. As a boundary condition, the pressure at the pipe outlet is $p = 96$ $[kPa]$. Water-air flow at ambient temperature was considered.

Figure 10 shows the average properties of the flow (bubble length, slug length, slug frequency and pressure) along the pipe.

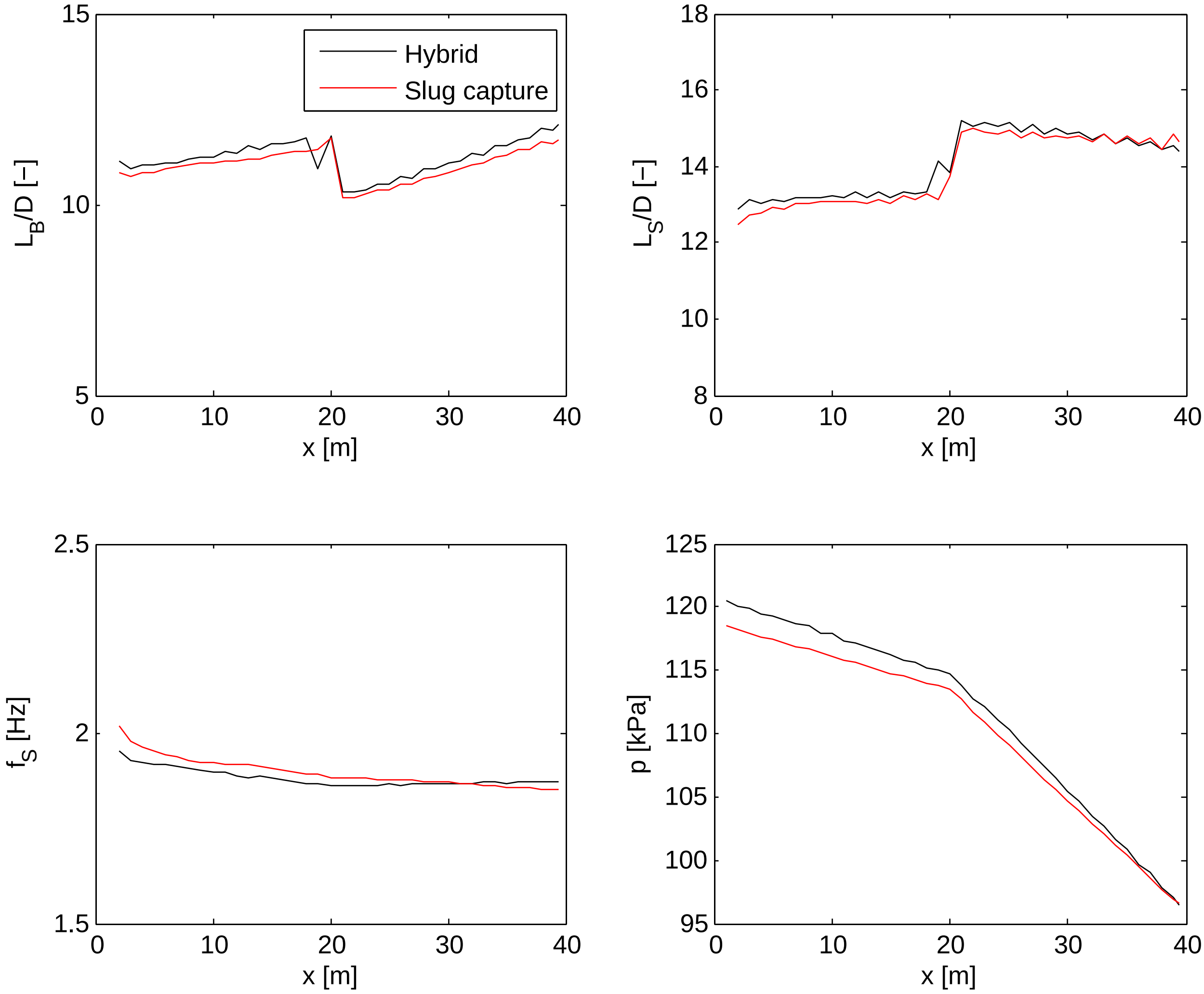
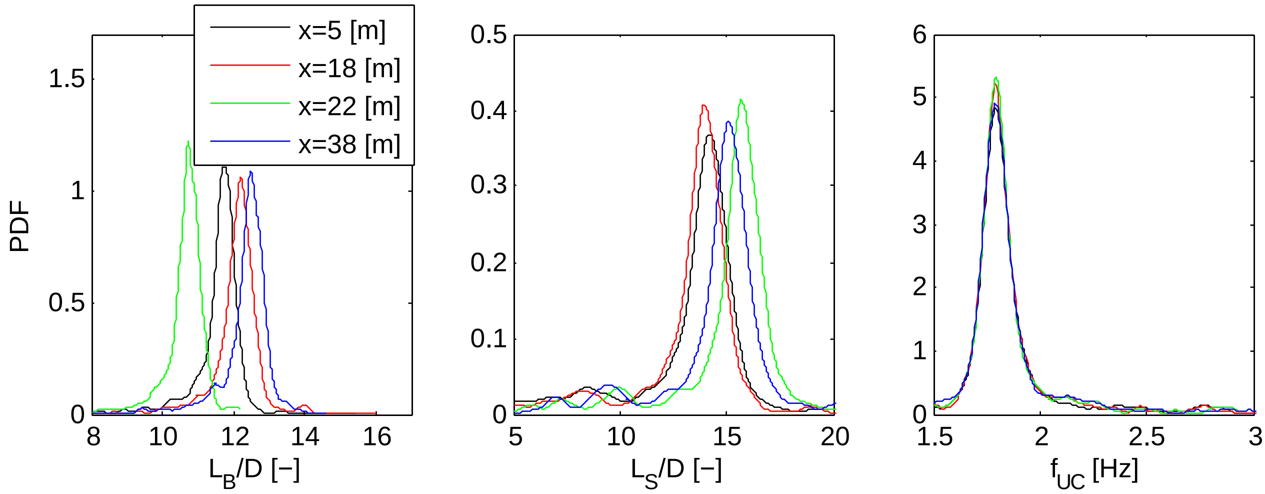


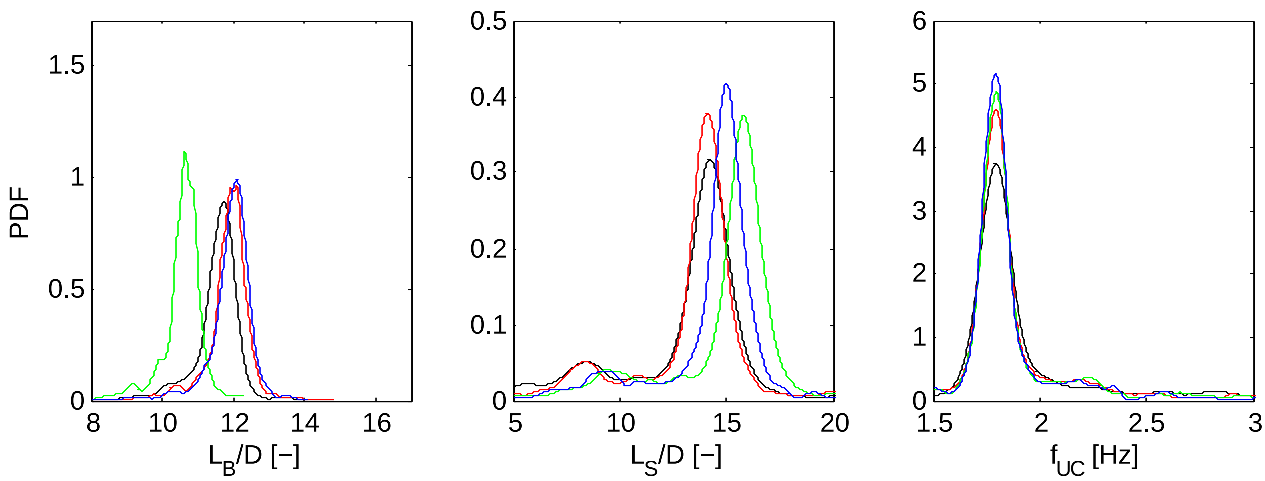
Figure 10: Average distribution of the flow properties along the pipe.

There was good agreement between the two codes. It was observed that for this pair of superficial velocities, bubble length increases along the pipe. It is important to remind that the liquid hold-up in the film is kept constant for the slug-tracking model. However, when the bubble passes throw the elbow, it suffers a significantly decrease. The slug length remains the same at the horizontal section, decreasing slightly along the inclined section. When it passes through the elbow, the slugs suffer a noticeable size increase. The slug frequency marginally declines along the pipe. The pressure gradient, however, reached a 13% discrepancy among the models. This inconsistency on the pressure gradient will be investigated in future works. Even though the pressure gradient showed a reasonable divergence, the results show that the hybrid methodology produces analogous results when compared to the slug capturing methodology.

Figure 11 displays the probability density functions (PDF) of some flow properties (bubble length, slug length, unit cell frequency and pressure) at $x = 5$, $x = 18$, $x = 22$ and $x = 38$. Figure 11a shows the PDF results of the hybrid code, and Fig. 11b shows the PDF results of the slug capturing code. The two models provided similar outputs.



(a) Hybrid code



(b) Slug capture code

Figure 11: PDF distribution at the virtual probes located at $x = 5.0$ $[m]$, $x = 18.0$ $[m]$, $x = 22.0$ $[m]$ and $x = 38.0$ $[m]$.

The PDF distributions bring similar conclusions as the average distributions. The two probes placed near the elbow, at $x = 18$ $[m]$ and $x = 22$ $[m]$, show how the elbow affects the bubbles crossing that position. The change of direction to upward inclination gives one additional force acting on the unit cell: the gravity. As a result of the gravity, a film crossing the elbow is expected to decelerate, and the amount of liquid being picked up from the slowly-moving film is added to the front of the slug. The unit cell frequency hardly changes along the four virtual probes.

Table 1 shows the time for simulation and the number of iterations. Both simulations stopped after 600 bubbles exited the pipe, starting to count after 80 $[s]$. The code was implemented in Fortran90, using object-oriented programming. A Intel(R) Core(TM) i7 4GHz computer performed the simulations.

Table 1: Computational cost for simulations

	Hybrid code	Slug capturing code
Simulation time [min]	4.122	9.027
Number of iterations	80,416	80,917

The hybrid code uses the slug tracking methodology to simulate the propagation of slugs. The hybrid code was faster since the slug tracking model has a very low computational cost when compared to the slug capturing model, as observed in Tab. 1.

4. CONCLUSIONS

A hybrid methodology to simulate slug flows in pipes was presented. The new approach integrated two models previously developed at NUEM: a slug capturing based on a Lagrangian two-fluid model, and a slug tracking model based on mass balance and momentum applied to the unit-cell structures. It was performed simulations with change of direction from horizontal to upward inclined pipes.

The results from the hybrid methodology were compared to those obtained from a slug capturing methodology. It was analyzed the bubble length, slug length, slug frequency, unit-cell frequency and pressure gradient. The models provided similar results for most of the parameters, except the pressure gradients, with a 13% discrepancy. Overall, the simulations have shown that there is a good agreement between the codes. Additionally, the use of the hybrid code reduced in 54% the computational cost of the simulations. For larger pipes, the reduction in the computational time is expected to be higher.

In future works, more simulations will be run, and the differences observed in the pressure gradients obtained from the two models will be investigated; results from the hybrid model will be compared to experimental data. It will be performed simulations with change of direction from horizontal to downward inclined, from downward inclined to horizontal and upward inclined to horizontal.

5. ACKNOWLEDGMENTS

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6. REFERENCES

Conte, M.G., 2014. *Estudo numérico e experimental da geração de golfadas em um escoamento bifásico de gás-líquido*. Master's thesis, Universidade Tecnológica Federal do Paraná, Curitiba. Programa de Pós-graduação em Engenharia Mecânica e de Materiais.

Conte, M.G., Cozin, C., Barbuto, F.A. and Morales, R.E.M., 2014. "A two-fluid model for slug flow initiation based on a lagrangian scheme". In *ASME 2014 4th Joint US-European Fluids Engineering Division Summer Meeting*. Chicago, Illinois, USA. 11 p.

Dukler, A.E. and Hubbard, M.G., 1975. "A model for gas-liquid slug flow in horizontal and near horizontal tubes". *Ind. Eng. Chem*, Vol. 14, No. 4, pp. 337–347.

Fernandes, R.C., Semiat, R. and Dukler, A.E., 1983. "Hydrodynamics model for gas-liquid-slug flow in vertical tubes". *AIChE Journal*, Vol. 29, No. 6, pp. 981–989.

Franklin, E.M. and Rosa, E.S., 2004. "Dynamic slug tracking model for horizontal gásliquid flow". In *3rd International Symposium on Two-Phase Flow Modelling and Experimentation*. Pisa. 8 p.

Ishii, M., 1975. *Thermo-Fluid Dynamic Theory of Two-phase Flow*. Eyrolles, Paris. 248 p.

Renault, F., 2007. *A Lagrangian slug capturing scheme for gas-liquid flows in pipes*. Ph.D. thesis, Norwegian University of Science and Technology Faculty of Engineering Science and Technology, Trondheim.

Rodrigues, H.T., 2009. *Simulação numérica do escoamento bifásico de gás-líquido no padrão de golfadas utilizando um modelo lagrangeano de seguimento de pistões*. Master's thesis, Universidade Tecnológica Federal do Paraná, Curitiba. Programa de Pós-graduação em Engenharia Mecânica e de Materiais.

Taitel, Y. and Barnea, D., 1990. "*Two-Phase Slug Flow*". *Advances in Heat Transfer*, Vol. 20, No. 1, pp. 83–132.

Taitel, Y. and Barnea, D., 1993. "*A model for slug length distribution in gas-liquid slug flow*". *Int. J. Multiphase Flow*, Vol. 19, No. 5, pp. 829–838.

Taitel, Y. and Barnea, D., 1998. "*Effect of gas compressibility on a slug tracking model*". *Elsevier*, Vol. 53, No. 11, p. 2089–2097.

Wallis, G.B., 1969. *One-dimensional two-phase flow*. McGraw-Hill, New York. 408 p.

Zuber, N. and Findlay, J., 1965. "Average volumetric concentration in two-phase flow systems". *J. Heat Transfer*, , No. 87, pp. 453–468.

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