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OPTIMIZATION OF THE INTERFACIAL SHEAR STRESS FOR THE SIMULATION OF HORIZONTAL VISCOUS OIL-GAS FLOWS WITH THE 1D TWO-FLUID MODEL

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Abstract: *The present work addresses the effect of interfacial shear on the simulation of viscous oil-gas flows with the 1D Two-Fluid Model. An optimization procedure is employed in order to develop new expressions for the interfacial friction factor based on experimental measurements of stratified flow cases. In such methodology, a steady-state fully developed version of the 1D Two-Fluid Model is used and good results are obtained in the optimization procedure of the new expressions by the model. They are then used for simulating slug and stratified flow cases through the numerical solution of the 1D Two-Fluid Model in fine meshes (Slug Capturing methodology). Results are explored via mesh convergence tests and analyzes of the influence of a direct increase of the interfacial shear on the results. The study reveals that expressions for the interfacial friction factor elaborated based on methodologies which use the steady-state fully developed 1D Two-Fluid Model are not necessarily directly applicable to the Slug Capturing framework, which intrinsically predicts the interfacial dynamics. Additional care must be taken due to ill-posedness of the model. Finally, it is concluded that future expressions for the interfacial friction factor should include probably include dynamic effects.*

Keywords: *Two-Phase Flow, Optimization, Viscous Oil, 1D Two-Fluid Model, Well-Posedness.*

1. INTRODUCTION

The ongoing decrease in the production of conventional oils in many traditional locations, like some fields in the Norwegian continental shelf, along with the strong increase in the world energy demand is impelling the oil companies to pay special attention, for example, to the production of heavy oils. Within its vast range of characteristics, the high dynamic viscosity of some oils present particular challenges for its transportation through long distances. When gas is present, it is very important to study the characteristics of the two-phase viscous oil-gas flows in order to ensure a proper design of pipelines and field operations. However, most of the simulation models to date were developed and validated for flows with low viscosity oils.

Several recent studies have addressed the particular characteristics of high viscosity oil-gas flows. Experiments in order to measure the holdup, pressure drop and to gather information about the flow patterns in the pipe have been carried out by various groups, as reviewed by Zhang *et al.* (2012). Gokcal (2008) and Zhao *et al.* (2013), for example, have investigated the characteristics of gas-oil two-phase flow in a horizontal pipe, with particular emphasis to slug flow. Eskerud Smith *et al.* (2011) performed experiments in a horizontal pipe with different oils (up to 100 cp) and dense gas (SF_6) in different flow regimes. Using their database, Khaledi *et al.* (2014) developed a steady state "Unit-Cell Model" model to reproduce the experimental results. Besides, Folletti *et al*. (2011) have performed experiments with air-high viscosity oils in a short horizontal pipe and Losi *et al*. (2016) have focused on the statistical properties of horizontal high viscosity oil-air slug flow.

Most modeling efforts to simulate viscous oil-gas flows have focused in Unit-Cell (Dukler & Hubbard, 1975) type of models. However, Pasqualette *et al*. (2015) and Ferrari *et al*. (2016) have investigated the performance of the Slug Capturing methodology (Issa & Kempf, 2003), i.e., the solution in fine meshes of the 1D Two-Fluid Model, for predicting the dynamic flow evolution and regime transition automatically within the model framework. In such approach, waves grow at the interface of stratified flow, eventually leading to large amplitude waves or slugs, which propagate through the pipe. However, according to Andritsos (1986) and Tzotzi & Andritsos (2013), stratified wavy flow of viscous oils may have different characteristics when compared to low viscosity oils. For this reason, different authors have tried to incorporate changes in the interfacial shear formulation, to account for the effect of oil viscosity explicitly (Andritsos & Hanratty, 1987; Andreussi & Persen, 1987; Spedding & Hand, 1997; Tzotzi & Andritsos, 2013; Zhao *et al.*, 2015). Furthermore, the critical velocity to large amplitude Kelvin-Helmholtz waves is usually lower than what it is observed for low viscosity oils and two-dimensional (2D) waves are not observed in experiments with viscous oils. Pasqualette *et al*. (2015) have observed that common formulations of the interfacial friction factor found in literature implemented in a Slug Capturing methodology can generate results with significant discrepancy to experiments.

The objective of the present work is to investigate the efficiency of new formulations for the interfacial shear stress, elaborated with the aid of an optimization methodology, on the simulations of horizontal viscous oil-gas flows by the numerical solution in fine meshes of the 1D Two-Fluid Model. The viscous oil-SF_6 experimental database of Eskerud Smith *et al.* (2011) is used in this work, obtained for a horizontal 68.6 mm ID pipe of 52.92 m long equipped with a gamma densitometer, which records the holdup signals at a certain position of the pipe. Firstly, the mathematical model is present, followed by the previously mentioned optimization procedure. In addition, the numerical methodology is briefly outlined, the numerical results and corresponding discussions are presented and the conclusions of the study are made.

2. MATHEMATICAL MODEL

The model used is based on the Two-Fluid Model (Ishii, 1975), i.e., on two sets of conservation equations separately formulated for each phase whose interaction is taken into account through interfacial transfer source terms. In this work, solely the hydrodynamics of the flow is of interest and the model is applied to laboratory-scale pipes. In field applications, pipelines might actually span several kilometers. Hence, the one-dimensional (1D) isothermal version of the Two-Fluid Model is used, targeting horizontal and nearly horizontal gas-liquid flows. In addition, the following hypotheses are considered in its formulation (Issa & Kempf, 2003; Pasqualette *et al.*, 2015): (*i*) the phases are fully continuous (bubbles and droplets are neglected); (*ii*) constant density of the liquid phase; (*iii*) ideal gas equation of state for the gas phase; (*iv*) constant viscosities of both phases; (*v*) negligible momentum distribution parameter effects; (*vi*) hydrostatic sectional pressure distribution. The mass conservation equations for the gas and liquid phases are given by Eqs. (1) and their corresponding momentum conservation equations by Eqs. (2) and (3).

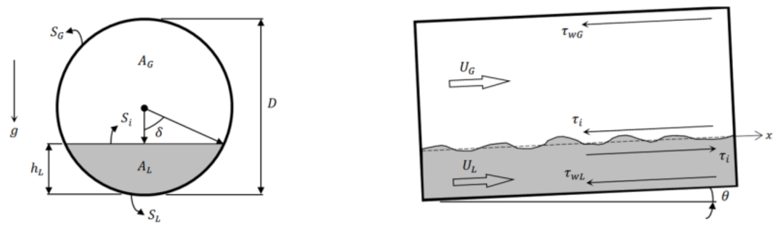
$$\frac{\partial(\alpha_G\rho_G)}{\partial t}+\frac{\partial(\alpha_G\rho_G U_G)}{\partial x}=0 \quad ; \quad \frac{\partial(\alpha_L\rho_L)}{\partial t}+\frac{\partial(\alpha_L\rho_L U_L)}{\partial x}=0 \tag{1}$$

$$\frac{\partial(\alpha_G\rho_G U_G)}{\partial t}+\frac{\partial\left(\alpha_G\rho_G {U_G}^2\right)}{\partial x}=-\alpha_G\frac{\partial P}{\partial x}-\alpha_G\rho_G g\left(\cos\theta\frac{\partial h_L}{\partial x}+\sin\theta\right)-\tau_{wG}\frac{S_G}{A}-\tau_i\frac{S_i}{A} \tag{2}$$

$$\frac{\partial(\alpha_L\rho_L U_L)}{\partial t}+\frac{\partial\left(\alpha_L\rho_L {U_L}^2\right)}{\partial x}=-\alpha_L\frac{\partial P}{\partial x}-\alpha_L\rho_L g\left(\cos\theta\frac{\partial h_L}{\partial x}+\sin\theta\right)+\alpha_L\sigma\frac{\partial^3 h_L}{\partial x^3}-\tau_{wL}\frac{S_L}{A}+\tau_i\frac{S_i}{A} \tag{3}$$

In the previous equations, the subscripts G and L stand for the gas and liquid phases. The variables ρ_k, α_k, U_k are the density, volume fraction (or holdup) and bulk velocity of phase k, respectively. P is the pressure (at the interface by the gas phase), θ is the pipe inclination angle (positive when the flow is upward and negative otherwise), h_L is the height of the liquid phase, x is the axial direction, g is the gravitational acceleration, A is the cross-sectional pipe area and σ is the gas-liquid surface tension (static interfacial tension). Lastly, τ_i is the interfacial shear stress, S_i is the interfacial perimeter, τ_{wk} is the shear stress between the wall and the phase k and S_k is the wall-wetted perimeter related to phase k. It is important to enhance that the surface tension term in Eq. is exclusive to stratified

It is important to enhance that the formulation for the surface tension term in Eq. (3) is exclusive to flow models, such as the one of this work, that use the stratified cross-sectional configuration assumption with a flat interface. The flow parameters and geometry is illustrated in Figure 1, where δ is half of the liquid phase wetted central angle, D is the piper inner diameter and A_k is the cross sectional area of the phase k.



(a) Pipe cross section. (b) Pipe longitudinal section.

Figure 1. Gas (white) and liquid (grey) flow geometry and its main parameters.

The geometrical parameters depicted in Figure 1 can all be calculated from δ, using straightforwardly obtained relations. As a consequence, during the solution of the model, δ has to be evaluated from α_L, which can be performed through the explicit approximated expression of Biberg (1999a) of Eq. (4).

$$\delta \approx \pi\alpha_L + \left(\frac{3\pi}{2}\right)^{1/3}\left[1 - 2\alpha_L + {\alpha_L}^{1/3} - (1-\alpha_L)^{1/3}\right] \tag{4}$$

The influence of the referred expression on the results of the model was properly addressed by Pasqualette *et al.* (2014), which compared it against the results obtained with the exact implicit geometric relation between δ and α_L.

3. OPTIMIZATION OF THE INTERFACIAL FRICTION FACTOR

The interfacial friction factor f_i is used in the evaluation of the interfacial shear stress through Eq. (5), which is a Fanning-like expression which uses the gas phase as reference due to the classical assumption in which the gas stream is treated as a single phase flow limited by a wall and a moving interface, i.e., the liquid phase.

$$\tau_i = \frac{1}{2} f_i\, \rho_G |U_G - U_L|(U_G - U_L) \tag{5}$$

As previously mentioned, one of the purposes of this work is to come up with a new expression for f_i based on the stratified flow experiments contained the in the viscous oil-SF_6 database of Eskerud Smith *et al.* (2011). Several key works (Andritsos & Hanratty, 1987; Andreussi & Persen, 1987; Spedding & Hand, 1997) have elaborated explicit empirical correlations for f_i based on stratified flow experimental data of integral flow measurements (liquid holdup and pressure gradient), where a proposed expression is fitted against reference values for the interfacial friction factor. Often labelled as "experimental", such values are calculated mostly by a steady-state fully-developed momentum balance at the gas phase with the aid of the experimental values of the liquid holdup and the pressure gradient and a correlation for the wall shear stress of the gas, when such measurements are not available (Newton & Behnia, 1998). However, in this work, rather than using a two-step procedure (calculation of the "experimental" values of f_i and then fitting the proposed expression), an optimization problem is formulated and solved employing measurements of the integral parameters directly in steady-state fully-developed momentum balances.

The optimization problem consists in finding directly the values of the adjustable coefficients of the proposed expression for f_i which, when used in a steady-state fully-developed Two-Fluid Model version, provide the closest results of liquid holdup and pressure gradient to the measurements. Such simple version of the Two-Fluid Model is represented by the momentum conservation equations of the gas and liquid phases submitted to the previously mentioned hypotheses and expressed by Eqs. (6). In these equations, $dP/dx|_k$ represents the pressure gradient in phase k.

$$\left.\frac{dP}{dx}\right|_G = -\rho_G g \sin\theta - \tau_{wG}\frac{S_G}{A_G} - \tau_i \frac{S_i}{A_G} \quad ; \quad \left.\frac{dP}{dx}\right|_L = -\rho_L g \sin\theta - \tau_{wL}\frac{S_L}{A_L} + \tau_i \frac{S_i}{A_L} \tag{6}$$

The simplified version of the Two-Fluid Model is solved by the procedure of Andritsos & Hanratty (1987), in which by a standard root-finding method, e.g. secant and bisection methods, the value of liquid holdup is evaluated by equalizing the pressure gradients in both phases. The objective function of the optimization problem consists in a combination, e.g. total sum and average, of the values of ϵ_j for each case (flow) j of the experimental database. The mentioned parameter is calculated via Eq. (7), in which, for a general variable ζ, $\zeta|_{\mathrm{calc},j}$ and $\zeta|_{\mathrm{exp},j}$ are the values of ζ calculated by the simple Two-Fluid Model and the experimental values, respectively.

$$\epsilon_j = \sqrt{\left(\frac{\alpha_L|_{\mathrm{calc},j} - \alpha_L|_{\mathrm{exp},j}}{\alpha_L|_{\mathrm{exp},j}}\right)^2 + \left(\frac{dP/dx|_{\mathrm{calc},j} - dP/dx|_{\mathrm{exp},j}}{dP/dx|_{\mathrm{exp},j}}\right)^2} \tag{7}$$

The solution of the optimization problem is performed by the standard version of the evolutionary Particle Swarm optimization algorithm (Kennedy & Eberhart, 1995). Only stratified flows with small amplitude waves were used in the optimization procedure for proposing a new expression for f_i. In the 35 cases chosen, the measured values of ρ_G/ρ_L vary between 0.05 and 0.06, μ_G/μ_L fluctuates between 1.3×10^{-4} and 1.8×10^{-4} while μ_L remain between 80 and 110 cp. Besides, the values of the superficial Reynolds number $\mathrm{Re}_{sk} = \rho_k U_{sk} D/\mu_k$ (U_{sk} is the superficial velocity) of phase k, vary between 610 and 720, for the liquid, and 2.5×10^5 and 1.9×10^6 for the gas.

As it is formulated, the optimization procedure allows the direct optimization f_i rather than of coefficients of an expression used for calculating f_i. This is useful, not only for providing the best possible results that the simple Two-Fluid Model can obtain for the selected cases, but also it serves as a way to discover the best expressions for the wall shear stresses τ_{wk} that suits the model. This analysis is not shown here due to space limitations. The selected wall friction factor correlations are described in the following topic.

3.1. Wall friction factor correlations

The wall shear stress τ_{wk} of each phase can be evaluated as a function of the Fanning wall friction factor f_k, as represented by Eq. (8).

$$\tau_{wk} = \frac{1}{2} f_k \rho_k |U_k| U_k \tag{8}$$

In the laminar regime, the friction factor $(f_G)_{\text{lam}}$ of the gas phase is calculated by the classical Hagen-Poiseuille expression represented in Eq. (9). Due to the high viscosity of the liquid phase, rather than also using the Hagen-Poiseuille expression for its laminar friction factor $(f_L)_{\text{lam}}$, an improved expression based on the analytical solution of Biberg (1999b), based on the parameter D_{hL}^*, which is a polynomial function of the angle δ is selected.

$$(f_G)_{\text{lam}} = \frac{16}{\text{Re}_G} \quad ; \quad (f_L)_{\text{lam}} = \frac{16}{\text{Re}_L} \frac{D_{hL}}{D_{hL}^*} \tag{9}$$

In both previous equations, the Reynolds number Re_k of phase k is calculated with its respective hydraulic diameter D_{hk} via the expression $\text{Re}_k = \rho_k U_k D_{hk}/\mu_k$. The hydraulic diameter of the gas phase is evaluated based on the previously mentioned hypothesis that it is flowing as in a closed conduit and that the liquid behaves as a “moving wall”: $D_{hG} = 4A_G/(S_G + S_i)$. For the liquid, the interface is classically interpreted as a free surface and its hydraulic diameter is evaluated as $D_{hL} = 4A_L/S_L$. In the turbulent regime, the analysis made via the optimization revealed that correlations for the wall friction factor that also account for f_i provide satisfactory results. Those interfacial friction factors refer to a scenario of smooth interface f_{i0}, and might be evaluated with the interfacial Reynolds number $\text{Re}_i = \rho_G |U_G - U_L| D_{hG}/\mu_G$. The Hagen-Poiseuille expression is employed for the laminar regime, $(f_{i0})_{\text{lam}}$, and a Blasius-type expression is chosen for the turbulent regime, $(f_{i0})_{\text{turb}}$ (Issa & Kempf, 2003). Those are shown in Eqs. (10).

$$(f_{i0})_{\text{lam}} = \frac{16}{\text{Re}_i} \quad ; \quad (f_{i0})_{\text{turb}} = 0.046 \text{Re}_i^{-0.2} \tag{10}$$

For the gas phase, the turbulent friction factor $(f_G)_{\text{turb}}$ is evaluated with the expression of Biberg (1998), formulated in Eq. (11), based on the absolute wall rugosity ε_G in the gas phase. This should take into account not only the effects of the absolute pipe wall rugosity ε, but also the presence of liquid droplets which linger in the pipe wall due to high viscosity of the oil (Eskerud Smith *et al.*, 2011). Khaledi *et al.* (2014) worked on this same viscous oil-SF_6 database and proposed a correction for the rugosity, ε_G, shown in Eq. (11), where $\mu_{L,0} = 1.75 \times 10^{-3}$ Pa. s is a reference liquid viscosity.

$$\frac{1}{\sqrt{(f_G)_{\text{turb}}}} = \frac{-3.6 \log\left[\frac{6.9}{\text{Re}_G} + \left(\frac{1}{3.7}\frac{\varepsilon_G}{D_{hG}}\right)^{1.11}\right]}{1 + 4\sqrt{f_{i0}}\frac{|U_G - U_L|}{U_G}\log_{10}\left(1 + \frac{S_i}{S_G}\right)} \quad ; \quad \varepsilon_G = \varepsilon\left\{1 + \exp\left[-100\left(\frac{\mu_{L,0}}{\mu_L}\right)^2\right]\right\} \tag{11}$$

For the turbulent liquid friction factor $(f_L)_{\text{turb}}$, the correlation of Nossen *et al.* (2000), formulated in Eq. (12), is used. The expression can be seen as an interpolation between the expressions of Haaland (1983) and Hand (1991) by using the weighting parameter ψ_t, expressed in Eq. (13), in which $\text{Fr}_{i0} = \tau_{i0}/[(\rho_L - \rho_G) g D \cos\theta]$ is the “smooth” interfacial Froude number. The “smooth” interfacial shear stress τ_{i0} is evaluated with f_{i0} by using an expression analogous to Eq. (5).

$$\frac{1}{\sqrt{(f_L)_{\text{turb}}}} = 6.178\psi_t(\alpha_L \text{Re}_{sL})^{0.0695} - 3.6(1-\psi_t)\log\left[\frac{6.9}{\text{Re}_L} + \left(\frac{1}{3.7}\frac{\varepsilon}{D_{hL}}\right)^{1.11}\right] \tag{12}$$

$$\psi_t = \tanh(2000 \cdot \text{Fr}_{i0})\frac{S_i}{S_L} \tag{13}$$

In addition, for evaluating f_k and f_{i0} in the transitional regime between the laminar and turbulent regimes, it was used a smooth interpolation function, which employs the limit values of the friction factors in each regime and also transition values of Re_k and Re_{i0}, besides the current ones.

3.2. Obtainance of new expressions for the interfacial friction factor

In this work, the new expressions to be proposed target the implementation in a transient 1D Two-Fluid Model. The representation of the effect of the interfacial waves on the flow has been the goal of various literature correlations for f_i (e.g. Andritsos & Hanratty, 1987; Ottens *et al.*, 2001; Tzotzi & Andritsos, 2013). The transitional gas superficial velocity $U_{sG,\text{t}}$ above which there should be an increase in f_i due to the influence of interfacial waves is an important parameter to be defined. The expression of Tzotzi & Andritsos (2013) for the transitional velocity between the 2D and the Kelvin-Helmholtz waves stratified regime was selected here to indicate when an increase in f_i should be considered. Actually, the high viscosity of the oil makes the value of $U_{sG,\mathrm{t}}$ be approximately null in most of the cases. Yet, it is important to define $U_{sG,\mathrm{t}}$ with the purpose of building a more general framework. Its expression is shown in Eq. (14), where the reference values $\rho_{G,\mathrm{ref}}$, $\rho_{L,\mathrm{ref}}$, σ_{ref} and $\mu_{L,\mathrm{ref}}$ correspond to air and water at 1 atm and 20°C.

$$U_{sG,\mathrm{t}} = \frac{1}{0.65}\left(\frac{\rho_{G,\mathrm{ref}}}{\rho_G}\right)^{0.5}\left(\frac{\rho_{L,\mathrm{ref}}}{\rho_L}\right)^{-0.5}\left(\frac{\sigma_{\mathrm{ref}}}{\sigma}\right)^{-0.33}\ln\left[\frac{1.39}{U_{sL}}\left(\frac{\mu_{L,\mathrm{ref}}}{\mu_L}\right)^{0.15}\right] \tag{14}$$

In the new expressions for f_{i0}, Capillary numbers are one of the parameters that account for the influence of the complex small-scale interfacial phenomena. Thus, as shown in Eq. (15), two definitions of the Capillary number were considered: an interfacial Capillary number Ca_i, defined by Biberg (1999c), and a Capillary number Ca_{sG} calculated with the gas superficial velocity and its transitional value $U_{sG,\mathrm{t}}$, defined by Eq. (14).

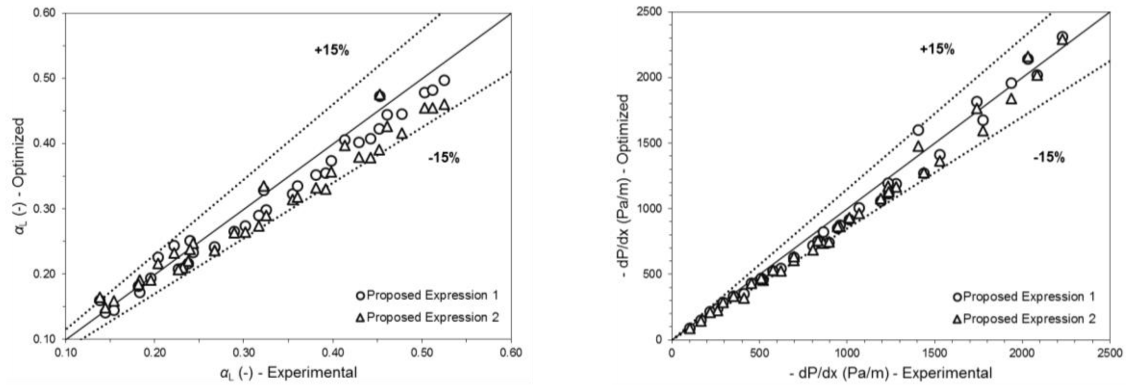
$$\mathrm{Ca}_i = \frac{\mu_m}{\sigma}\left|\frac{U_{sG}}{\epsilon_G^*} - \frac{U_{sL}}{\epsilon_L^*}\right| \quad ; \quad \mathrm{Ca}_{sG} = \frac{\mu_m}{\sigma}\left(U_{sG} - U_{sG,\mathrm{t}}\right) \tag{15}$$

For both definitions, a mixture viscosity μ_m is used, which might be calculated as $\mu_m = \mu_G\mu_L/(\gamma_L^*\mu_G + \gamma_G^*\mu_L)$ (Biberg, 1999c). The values of the parameters ϵ_G^*, ϵ_L^*, γ_G^*, γ_L^* are calculated via polynomial expressions found in Biberg (1999b; 1999c).

As the target of the efforts here is to correctly express how f_i should increase in relation to the smooth interfacial friction factor f_{i0} due to the interfacial dynamics, it is better to formulate new expressions for f_i/f_{i0} rather than solely f_i. This was inspired by Biberg (1999c) and presents an alternative to create correlations for f_i/f_G (e.g. Andritsos & Hanratty, 1987; Andreussi & Persen, 1987) and f_i/f_{sG} (Spedding & Hand, 1997), where f_{sG} is the friction factor of the gas evaluated with Re_{sG} rather than Re_G. It is important, then, to define the Standard Expression (Std) for the interfacial friction factor in Eq. (16), which serves as a benchmark for the simulations performed in this work. Finally, two new expressions for f_i/f_{i0} are elaborated, which are formulated in Eqs. (16) and labelled as Proposed Expression 1 (PE1) and Proposed Expression 2 (PE2), respectively.

$$\left.\frac{f_i}{f_{i0}}\right|_{\mathrm{Std}} = 1 \quad ; \quad \left.\frac{f_i}{f_{i0}}\right|_{\mathrm{PE1}} = 1 + \phi_1 \mathrm{Ca}_i{}^{\phi_2}\left(U_{sG} - U_{sG,\mathrm{t}}\right)^{\phi_3} \quad ; \quad \left.\frac{f_i}{f_{i0}}\right|_{\mathrm{PE2}} = 1 + \chi_1 \mathrm{Ca}_{sG}{}^{\chi_2}\exp\left(\chi_3\frac{h_L}{D}\right) \tag{16}$$

In the Proposed Expression 1, the obtained optimized values of the three coefficients ϕ_i's are $\phi_1 = 150$ (this coefficient is actually dimensional, however, for simplicity, its unit was dropped), $\phi_2 = 0.65$ and $\phi_3 = 0.38$. In the proposed expression 2, the transitional gas superficial velocity suggested by Newton *et al.* (1999) and the results obtained by Pasqualette *et al.* (2015) inspired the insertion of the exponential term. The original value of $\chi_3 = 3.1$ was kept, and therefore, solely $\chi_1 = 993$ and $\chi_2 = 1.29$ were obtained through optimization. The optimized average values of ϵ_j, Eq. (7), were 0.11 and 0.14 for the models 1 and 2, respectively, and the optimized liquid holdup and pressure gradient for the 35 selected cases are presented in . It can be seen in Figure 2 that the optimized results were indeed satisfactory, with deviations from the measurements below 15%, with slightly better results of the proposed expression 1.



(a) Liquid holdup. (b) Pressure Gradient.

Figure 2. Comparison between the experimental and optimized results for the 35 stratified wavy flow cases.

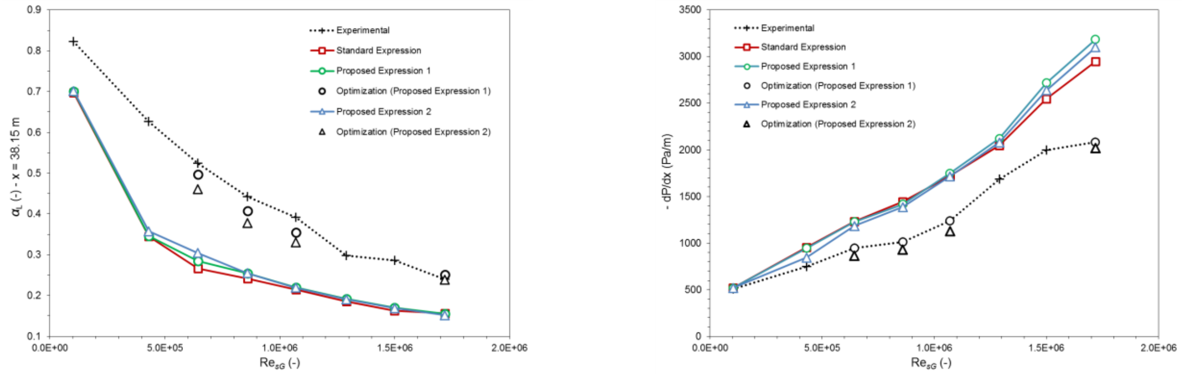
4. NUMERICAL METHODOLOGY

For the numerical solution of the Two-Fluid Model equations, Eqs. (1)-(3), the Finite Volume method was employed (Patankar, 1980). In the time integration of the conservation equations, a fully implicit first-order Euler scheme was used and, for the convection term, a high-order Total Variation Diminishing (Harten, 1983) scheme with the UMIST flux limiter function (Lien & Leschziner, 1994) was employed. The pressure is determined through a global continuity equation, obtained by the addition of the mass conservation equation of each phase normalized by their respective densities in a reference condition. The discretized equations are sequentially solved with an adaptation of the PRIME algorithm for handling the pressure-velocity coupling (Carneiro *et al.*, 2011; Nieckele *et al.*, 2013 and Simões *et al.*, 2014). The TDMA algorithm (Patankar, 1980) was used for solving the algebraic system formed by the discretized equations. In addition, a CFL number of 0.05 was defined. The simulation time for every simulation was of 500 seconds, for guaranteeing a steady solution. Lastly, at each time step, the maximum residue of all equations must be smaller than 10^{-5} for obtaining convergence.

5. RESULTS AND DISCUSSIONS

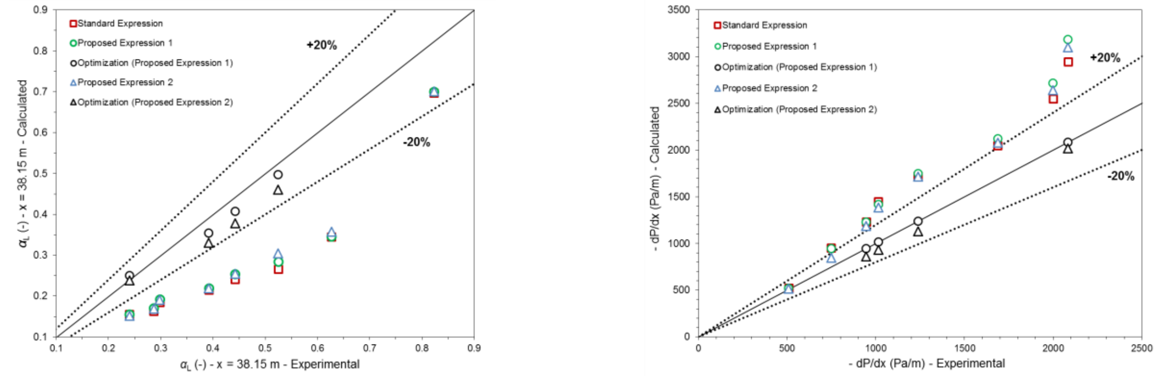
For testing the efficiency of the proposed interfacial friction expressions 1 and 2, eight cases in the viscous oil-SF_6 horizontal flows database of Eskerud Smith *et al.* (2011) were selected and simulated by the numerical solution of the Two-Fluid Model. Results were compared with the experimental measurements and the ones obtained with the standard interfacial friction expression. In these cases, Re_{sL} and Re_{sG} varied between 470 and 610 and 1.0×10^5 and 1.7×10^6, respectively. Besides, μ_G/μ_L varied between 1.3×10^{-4} and 1.7×10^{-4}, ρ_G/ρ_L fluctuated between 0.051 and 0.054, the values of μ_L were kept between 86 and 110 cp and $\sigma = 0.02$ N/m. In these eight cases, the flow patterns of slug and of stratified wavy with small and large amplitude waves are present, from which four cases were also used in the optimization procedure for obtaining the new expressions for f_i.

Based on previous works and in order to achieve a good compromise between computational effort and accuracy, a grid aspect ratio of $\Delta x/D = 0.75$ was chosen for all cases. Simulation of all eight cases using the standard and the new expressions for the interfacial friction factor were performed. The numerical mean liquid holdup, at $x = 38.15$ m, and the pressure gradient are compared against each other and against experimental values. Results of the optimization procedure, corresponding to stratified wavy flow cases with small amplitude, are also compared. Figures 3a and 3b show the liquid holdup and the pressure gradient, respectively, as a function of Re_{sG}, and Figures 4(a) and 4(b) compares calculated with experimental values.



(a) Liquid holdup. (b) Pressure gradient.

Figure 3. Comparison of the results of numerical simulation against experimental data and optimized values.



(a) Liquid holdup. (b) Pressure gradient.

Figure 4. Comparison of the results of numerical simulation against experimental data and optimized values.

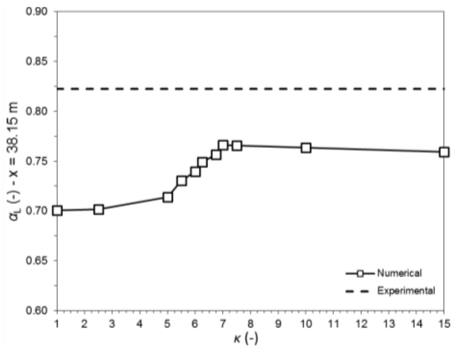
It can be first observed that the values of liquid holdup and pressure gradient obtained with the proposed expressions are similar to the results originated from the standard expression, which, with the exception of the case with smaller value of Re_{sG}, deviate more than 20% from the experimental data. It is also interesting to note that, for the cases which were used in the procedure that optimized the coefficients of the new expressions for f_i, the numerical results for mean hold-up and pressure drop with the transient model in a fine mesh are different from the ones obtained in the referred procedure, i.e., using a steady state momentum balance.

Analyzing the results, it is evident that the new expressions did not manage to improve the results obtained with the standard expression. The new expressions provide different qualitative behavior for the flow, intrinsically capturing waves and slugs. Still, they provided similar average results, which probably excludes the possibility of a poor choice of hydrodynamic parameters to represent the increase in f_i/f_{i0} . One can argue that the unexpected similarity of the results is a consequence of only employing the small amplitude waves in the optimization process. In other words, the values of f_i evaluated during the simulation are just not high enough and/or are very close to unity, not improving the prediction of liquid holdup and pressure gradient.

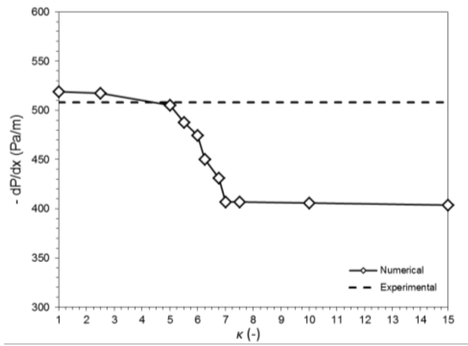
For addressing this point properly, the proposed expression 1 is modified by the introduction of a multiplying factor κ, which is shown in Eq. (17).

$$\frac{f_i}{f_{i0}} = 1 + \kappa\, \phi_1 \mathrm{Ca}_i{}^{\phi_2} \left(U_{sG} - U_{sG,\mathrm{t}}\right)^{\phi_3} \tag{17}$$

Figure 5 shows the influence of the multiplying factor in the liquid holdup and pressure gradient, for the slug case. At the same figure, the experimental values are indicated. It can be seen that, as the interface friction factor is increased through the κ factor, there is an improvement in the liquid holdup prediction, and at the same time, the pressure gradient prediction deteriorates for κ in the range from 5 to 6. For for $\kappa > 6.75$ the solution becomes stable, and the factor does not influence the prediction.



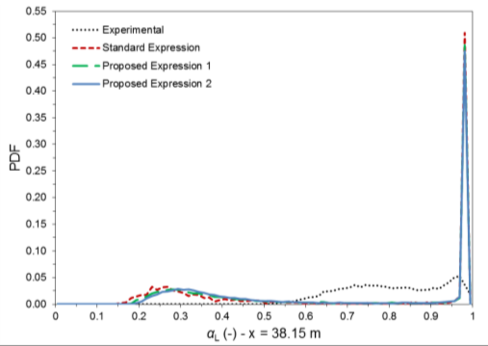
(a) Liquid holdup.



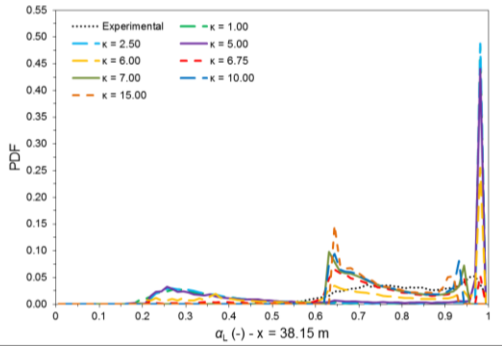
(b) Pressure gradient.

Figure 5. Integral results behavior of the slug flow case with the value of κ in the modified Proposed Expression 1.

To better understand the influence of κ parameter in the flow prediction, it is convenient to analyze the PDF's of the liquid holdup in the gamma densitometer position. Figure 6a compares the resulting PDF's of the simulations with the standard and proposed expression 1 (with κ =1) and 2, against the experimental data. As expected all numerical predictions are similar, showing deviations from the measurements. At Figure 6b the influence of κ in the PDF's are shown, where, it is evident that increasing values of f_i tend to stabilize the flow, diminishing the number of slugs, and to increase the holdup of the liquid film, which, as noticed in Figure 5, was originally much lower than the experimental data indicate it should be. For $\kappa > 6.75$, the stabilization is so strong that the slugs disappear, which justifies the equal stabilization of the integral results in Figure 5. For the previously mentioned values of $\kappa = 5.0$ and $\kappa = 6.0$, indeed, numerical PDF's is closer to the experimental PDF.



(a) Standard and Proposed Expressions.

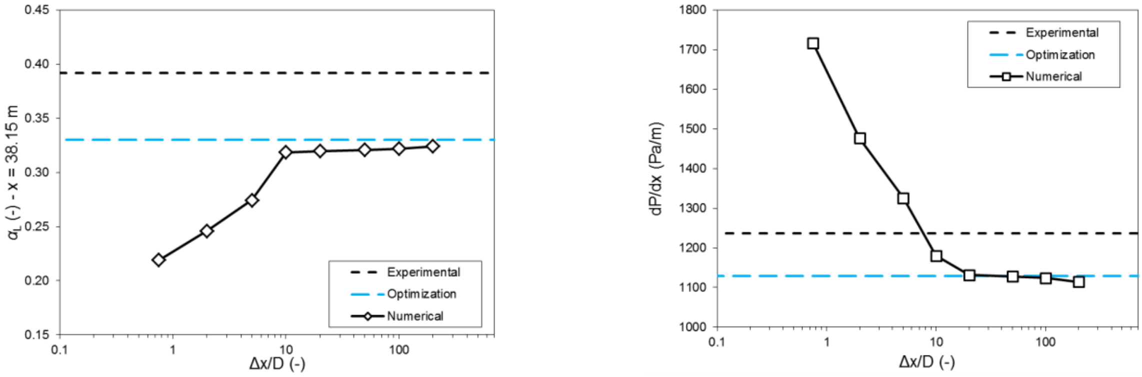


(b) Influence of κ in Proposed Expression 1.

Figure 6. Comparison of the experimental and numerical liquid holdup PDF's for the slug flow case.

As shown in Figure 6, although some values of κ are capable of slightly improving the numerical results, they are still very far from being satisfactory, especially because it fails to provide a good numerical interfacial dynamics behavior. Since the model used is not correctly representing the dynamics of the slug and waves formation, it indicates that the new expressions for the interfacial friction factor based on a steady-state fully-developed version of the Two-Fluid Model might not be the best way for improving the results of the viscous oil-SF_6 flows of Eskerud Smith *et al.* (2011), using a transient model which captures waves and slugs. In other words, dynamic values should be included in the estimation of the interfacial friction factor or the interfacial shear stress, as it will be explored in future works.

Another point that must addressed is the difference between the values of liquid holdup and pressure gradient obtained from the optimization, which are satisfactory, and from the numerical solution of the Two-Fluid Model for the stratified (small amplitude) wavy flows. Due to the high differences in the bulk velocities of gas and liquids in stratified wavy flows, it is possible that the model is ill-posed. Therefore, a mesh convergence test is performed for one of the stratified wavy flow case with small-amplitude interfacial waves, which was also used in the optimization procedure. In such test, the proposed expression 2 was employed for f_i and Figures 7 and 8 illustrate the behavior of the integral results of liquid holdup, pressure gradient and liquid holdup PDF's, respectively, as a function of the mesh aspect ratio $\Delta x/D$. A comparison with the experimental and optimization values is also included.



(a) Liquid holdup. (b) Pressure gradient.

Figure 7. Behavior of the integral results with Δx/D for a stratified small-amplitude waves (the proposed expression 2 was used for f_i).

By examining Figs. 7 and 8, one can promptly notice that, as expected, larger values of the mesh size tend to bring the average values closer to the optimization results, which are in better agreement with the experimental data. Nevertheless, as the value of $\Delta x/D$ is decreased and dynamic effects are now captured by the numerical solution of the Two-Fluid Model, not only physical but also spurious waves with very large amplitudes, i.e., high growth rates, appear. Such effects increase the numerical values of pressure gradient and decrease the mean liquid hold-up, as shown in Figure 7. Considering the intimate relation between the appearance of spurious waves, their growth rates and the ill-posedness of the model (Ramshaw & Trapp, 1978; Prosperetti & Tryggvason, 2007; Fullmer *et al.*, 2014), it is evident that this analyzed case is probably not well-posed. This should also be true for the other stratified wavy flow cases in which the optimization and numerical results differed. Therefore, the well-posedness and the hyperbolicity (Prosperetti & Tryggvason, 2007) of the model for the viscous oil-SF_6 flows of Eskerud Smith *et al.* (2011) should also be properly addressed.

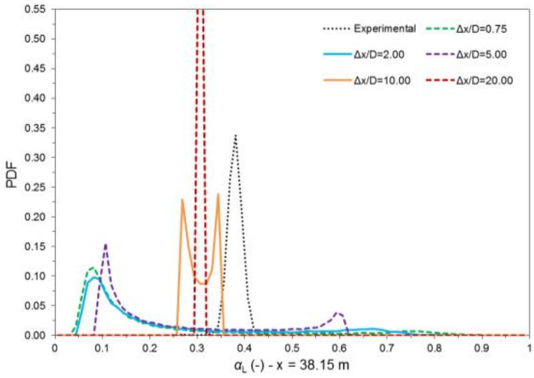


Figure 8. Behavior of the liquid holdup numerical PDF with $\Delta x/D$.

By combining the conclusions of the two previous analyzes, is can be noticed that special care must be taken when developing (or using) friction factor correlations, elaborated with methodologies based on steady-state fully-developed version of the Two-Fluid Model. This is especially true when evaluating modifications to the transient Two-Fluid Model in high resolution meshes, in order to better represent the interfacial dynamics in the referred cases, because the stratified wavy flows may be ill-posed. Without having to use known strategies for solving the ill-posedness of the Two-Fluid Model, e.g. addition of an artificial viscosity (Holmas *et al.*, 2008) or addition of an interfacial pressure (Stuhmiller, 1977), the interfacial shear stress should be represented in a way that simultaneously improves the representation of the interfacial dynamics of the flow and the hyperbolicity of the model. This should be accomplished, by the introduction of dynamic terms in the expression of f_i or τ_i, which will be the subject of future works.

6. CONCLUSIONS

New expressions for the interfacial friction factor were elaborated and adjusted based on an optimization procedure which used the experimental data of stratified (small amplitude) wavy flows of the horizontal viscous oil-SF_6 flows obtained by Eskerud Smith *et al.* (2011). Good agreement was obtained for steady state predictions of hold-up and pressure drop. Such correlations were also used with the purpose of improving the predictions of the numerical solution of the Two-Fluid Model in fine meshes for slug and stratified flows of the same database. However, the proposed expressions 1 and 2 provided results similar to the standard expression, for mean liquid holdup and its PDF and for the pressure gradient. In addition, differences between the results originated from the optimization procedure and from the numerical simulations of the transient model using fine meshes were observed even for stratified (small amplitude) wavy flows.

It was concluded that expressions for f_i elaborated from methodologies employing an optimization procedure, based on steady-state and fully-developed version of the Two-Fluid Model are not capable of correctly representing the interfacial dynamics for the viscous oil-SF_6 flows studied. The second analysis revealed that such drawback of the interfacial friction factor correlations may be induced by ill-posedness for stratified flow cases, i.e., when the velocity difference between the phases is high. Therefore, the development of a new expression for f_i should be capable to take into account the effects of interfacial dynamics as well as dealing with occasional loss of hyperbolicity of the model. This can be solely accomplished by the use of dynamic variables in the expression for the interfacial shear, which will be addressed in future works.

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