Severe slugging: modeling, simulation and stability criteria

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Outline

- Introduction
- Air-water model
- Simulation results
- Linear stability analysis
Some information

• Petroleum production in Brazil (2013): 985.5 million barrels of petroleum and other liquids (730 million barrels was crude oil, and 192.4 million barrels was biofuels) and 25.8 billions of cubic meters of natural gas.

• More than 91% of Brazil's oil production is offshore in very deep water and consists of mostly heavy grades, of which 79% was developed near the state of Rio de Janeiro, primarily in the Santos and Campos basins.

• Pre-salt (1000-2000 m water layer, 4000-6000 m under seabed) accounts for 15% of total oil production. Estimated pre-salt reserves: over 50 billion barrels of oil.

• Transported fluid is a multiphase, multicomponent mixture of oil, gas and water.

• Possible formation of emulsions, hydrates and wax.

• Modeling and simulation is very complex (flow assurance).
Offshore petroleum production systems
Offshore petroleum production systems
Motivation

• 2005: First contract between USP and Petrobras in the area of severe slugging in catenary risers.
• Analysis of experimental results in an air-water rig, made by CALTec (UK) under request of Petrobras, related to the influence of separator pressure in catenary riser (Wordsworth et al., 1998).
• Three main areas: modeling and simulation, stability analysis and multiphase rig.
Stationary state

Taitel (1986)
Slug formation

Taitel (1986)
Slug production

Taitel (1986)
Blowout
Severe slugging cycle

Schmidt (1977)
Severe slugging

• Terrain dominated phenomenon, characterized by the formation and cyclical production of long liquid slugs and fast gas blowdown.

• Instability that may appear for low gas and liquid flow rates.

• Main issues related to severe slugging are:
  - High average back pressure at well head, causing tremendous production losses.
  - High instantaneous flow rates, causing instabilities in the liquid control system of the separators and eventually shutdown.
  - Reservoir flow oscillations.
Production: 2400 STB/d. Mean pressure at “steady” state: 700 psi. Mean pressure with SS: 900 psi. Production loss: 800 STB/d.
Severe slugging classification

- **SS1**: the liquid slug length is greater to or equal to one riser length and maximum pipeline pressure is equal to the hydrostatic head of the riser (neglecting friction pressure drop).
- **SS2**: the liquid length is less than one riser length, with intermittent gas penetration at the bottom of the riser.
- **SS3**: there is continuous gas penetration at the bottom of the riser; visually, the flow in the riser resembles normal slug flow, but pressure, slug lengths and frequencies reveal cyclic variations of smaller periods and amplitudes compared to SS1.
- **OSC**: there are cyclic pressure fluctuations without the spontaneous vigorous blowdown.
Severe slugging mitigation devices

- SS mainly controlled by dydrostic pressure drop at the riser and compressibility at the pipeline. Mitigation devices:
  - Pressure increase at the separator: compresses the gas phase and makes the system less compressible. Disadvantage: reduction in production, as pressure difference between the reservoir and separator is the driving force.
  - Flow restriction in the riser (choke valve): in addition to the system pressure increase, any increase in flow rate at the top of the riser gives rise to an increase in frictional pressure drop across the valve, causing an additional stabilization effect. Reduction in production. System control action.
  - Gas injection: aeration of the liquid column and reduction of the pressure gradient. Relatively large gas flow rates are necessary. Pumping operational cost can be very significant.
  - Self-lifting (Tangesdall et al., 2002, 2003): gas transfer from the pipeline to the riser, without pumping power.
  - Venturi (Almeida & Gonçalves, 1999): flow acceleration at the pipeline, eliminating the stratified flow pattern.
Air-water model

• Model:
  - One-dimensional, isothermal flow and a mixture momentum equation in which gravitational and frictional terms are important.
  - Inertia and propagation of pressure waves were neglected both in the pipeline and riser (no-pressure-wave, NPW, approximation); pressure changes are felt instantaneously at any point.
  - Constant void fraction and pressure drop at the pipeline (taken from stationary state, stratified flow pattern).

• Improvements:
  - Inertial terms taken into account by using the rigid water-hammer approximation (not shown here).
  - Variable void fraction and pressure drop at the pipeline (not shown here).
  - Gas injection at the bottom of the riser.
  - Choke valve at the top of the riser.
Pipeline equations

- Condition $x > 0$:

\[ j_{gb} = 0 \]
\[ \frac{dx}{dt} = \frac{j_{i0} - j_{ib}}{\alpha_p} \]
\[ \frac{dP_g}{dt} = -P_g \left( j_{ib} - j_{i0} \right) + \frac{T_g}{T_0} P_0 j_{g0} \frac{1}{(L - x) \alpha_p + L_e} \]

\[ j_{i0} = \frac{Q_{i0}}{A} \]
\[ j_{g0} = \frac{R_g T_0 \dot{m}_{g0}}{P_0 A} \]
\[ L_e = \frac{\nu_e}{A} \]

\[ P_b = P_g + \rho_l g x \ \text{sen} \beta \]
Pipeline equations

- Condition $x = 0$:

\[
\dot{j}_{lb} = \dot{j}_{l0}
\]

\[
\frac{dP_g}{dt} = -P_g \dot{j}_{gb} + \frac{T_g}{T_0} P_0 \dot{j}_{g0} \frac{L \alpha_p + L_e}{L \alpha_p + L_e}
\]

\[
P_b = P_g
\]
Pipeline commutation

- Commutation from \( x = 0 \) to \( x > 0 \) :
  
  \[
  j_{gb}^- = j_{gb}^+ = 0 \quad j_{lb}^+ = j_{lb}^-
  \]
  \[
  \frac{dx^+}{dt} = 0
  \]

- Commutation from \( x > 0 \) to \( x = 0 \) :
  
  \[
  j_{gb}^+ = -\alpha_p \frac{dx^-}{dt} \quad j_{lb}^+ = j_{lb}^- + \alpha_p \frac{dx^-}{dt}
  \]
  \[
  \frac{dx^+}{dt} = 0 \quad j_{gb}^- = 0
  \]

- Commutations guarantee continuity in pressure and time derivative of pressure at the bottom of the riser.
Pipeline local equilibrium condition

- Momentum balance in stratified flow (Taitel, 1976) for stationary state:

\[
\tau_w g \frac{S_g}{\alpha_p} - \tau_w l \frac{S_l}{1 - \alpha_p} + \tau_i S_i \left( \frac{1}{1 - \alpha_p} + \frac{1}{\alpha_p} \right) + (\rho_l - \rho_g) A g \sin \beta = 0
\]
Riser equations

- Continuity:
  \[- \frac{\partial \alpha}{\partial t} + \frac{\partial j_l}{\partial s} = 0 \quad \frac{\partial}{\partial t} (P\alpha) + \frac{\partial}{\partial s} (P j_g) = 0\]

- Mixture linear momentum:
  \[\frac{\partial P}{\partial s} = -\rho_m \left( g \, \text{sen} \theta + 2 \frac{f_m}{D} j \, |j| \right)\]
  \[\rho_m = \rho_l (1 - \alpha) + \rho_g \alpha \quad \rho_g = \frac{P}{R_g T_g}\]

- Closure law (drift flux model):
  \[j_g = u_g \alpha = \alpha (C_d j + U_d)\]
  \[j_l = j - j_g = u_l (1 - \alpha) = (1 - \alpha C_d) j - \alpha U_d\]
Characteristic values

• For a general drift flux correlation:

\[ C_d = C_d (\alpha, j_g, j_l, P, \theta) \]

\[ U_d = U_d (\alpha, j_g, j_l, P, \theta) \]

• Characteristic direction for the NPW approximation is:

\[ e_1 = \frac{\partial j_g}{\partial \alpha} \]

• If the drift coefficients are independent of void fraction:

\[ \frac{\partial j_g}{\partial \alpha} = \frac{j_g}{\alpha} = u_g \]
Drift flux correlation

• Bendiksen correlation (1984): \( Fr_j = \frac{j}{\sqrt{g \cdot D}} \)

\[ C_d = \begin{cases} 
1.05 + 0.15 \sin \theta & \text{for } Fr_j < 3.5 \\
1.2 & \text{for } Fr_j > 3.5 
\end{cases} \]

\[ U_d = \begin{cases} 
\sqrt{g \cdot D} (0.35 \sin \theta + 0.54 \cos \theta) & \text{for } Fr_j < 3.5 \\
0.35 \sqrt{g \cdot D} \sin \theta & \text{for } Fr_j > 3.5 
\end{cases} \]

• Characteristic equations become:

\[ \frac{D_g \alpha}{Dt} + \alpha \frac{\partial}{\partial s} \left( C_d \cdot j + U_d \right) - \frac{\partial j}{\partial s} = 0 \]

\[ \alpha \frac{D_g P}{Dt} + P \frac{\partial j}{\partial s} = 0 \]

\[ \frac{D_g}{Dt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial s} \]
Riser, gas lift

- Gas injection aerates the liquid column, reduce the pressure gradient and can stabilize the flow in severe slugging, although relatively large gas flow rates are necessary. Gas injection at position $S_l$ introduces discontinuities in position in the gas superficial velocity and void fraction:

$$j_g^+ = j_g^- + \frac{T_g}{T_0} \frac{P_0}{P_l} j_{g0l} \quad \alpha^+ = \frac{j_g^+}{(C_d j + U_d)^+}$$

$$j_{g0l} = \frac{R_g T_0 \dot{m}_{gl}}{P_0 A}$$
Riser, choke valve

• In normal operation in petroleum production systems the choke valve controls the flow. Choking can stabilize the flow by increasing the back pressure. For low pressures, typical of air-water laboratory systems, the valve operates in subcritical condition.

\[ P_t - P_s = K_v \frac{1}{2} \rho_m t |j_t| \]

(homogeneous)

\[ P_t - P_s = C j_{lt} |j_{lt}| \]

(Jansen et al., 1996)
Pipeline-riser coupling

\[ P(s = 0, t) = P_b(t) \]
\[ j_g(s = 0, t) = j_{gb}(t) \quad j_l(s = 0, t) = j_{lb}(t) \]
\[ \alpha(s = 0, t) = \alpha_b(t) = \frac{j_{gb}}{C_{db}j_b + U_{db}} \]

Piping state variables: (constant \( \alpha_p \)):
\( x(t), P_g(t) \)

Riser state variables:
\( P(s, t), \alpha(s, t), j(s, t) \)

\( \dot{m}_g_0, Q_{l0} \) (inputs)

\( P_s \) (input)
Stationary state

- The stationary state, when stable, is the steady state flow operational regime. It is used as the initial condition for the transient simulations and also as the base solution for the linear stability analysis.

- For the pipeline:
  \[
  \tilde{J}_{lb} = J_{l0} \\
  \tilde{J}_{gb} = \frac{T_g}{T_0} \frac{P_0}{\tilde{P}_b} J_{g0}
  \]

- For the riser:
  \[
  \tilde{J}_l = J_{l0}
  \]

  \[
  \tilde{J}_g = \begin{cases} 
  \frac{T_g}{T_0} \frac{P_0}{\tilde{P}} J_{g0} & \text{for } s < s_l \\
  \frac{T_g}{T_0} \frac{P_0}{\tilde{P}} (J_{g0} + J_{g0l}) & \text{for } s > s_l 
  \end{cases}
  \]
Stationary state

• For the riser (continued):

\[
\tilde{\alpha} = \begin{cases} 
\frac{\tilde{\alpha}}{\tilde{\alpha}} = \frac{\tilde{j}_{g0}}{\tilde{C}_d \tilde{j}_{g0} + (\tilde{j}_{l0} + \tilde{U}_d) \frac{T_0}{T_g} \frac{\tilde{P}}{P_0}}{} 
& \text{for } s < s_l \\
\frac{\tilde{j}_{g0} + \tilde{j}_{g0l}}{\tilde{C}_d (\tilde{j}_{g0} + \tilde{j}_{g0l}) + (\tilde{j}_{l0} + \tilde{U}_d) \frac{T_0}{T_g} \frac{\tilde{P}}{P_0}}{\tilde{P}} 
& \text{for } s > s_l
\end{cases}
\]

\[
\frac{\partial \tilde{P}}{\partial s} = -\tilde{\rho}_m \left( g \sin \theta + 2 \frac{\tilde{f}_m}{D} \tilde{j}^2 \right)
\]
Experiments

Wordsworth et al. (1998) for catenary risers:

\[ Z = 9.886 \, \text{m} \]
\[ X = 6.435 \, \text{m} \]
\[ s_t = 12.5463 \, \text{m} \]

\[ D = 5.25018 \times 10^{-2} \, \text{m} \]

Figure 1.10: Schematic of the test facility (Wordsworth et al., 1998).
Experiments

(a) Inlet flows.
(b) Pressures.

Figure 1.11: Experimental history for SS1 (case 2, Table 1.3).
Experiments

Figure 1.12: Experimental history for SS2 (case 7, Table 1.4).
Experiments

Figure 1.13: Experimental history for SS3 (case 11, Table 1.3).
Experiments

(a) Inlet flows.

(b) Pressures.

Figure 1.14: Experimental history for OSC (case 17, Table 1.3).
Experiments

Figure 1.15: Experimental history for slug flow (case 14, Table 1.3).
Experiments

(a) Inlet flows.  

(b) Pressures.

Figure 1.16: Experimental history for plug flow (case 16, Table 1.3).
Experiments

(a) Inlet flows.

(b) Pressures.

Figure 1.17: Experimental history for bubbly flow (case 20, Table 1.4).
Experiments

Figure 1.22: Experimental history for case 4, Table 1.3.

Controlability of the boundary conditions!
Severe slugging pattern recognition

Figure 1.18: FFT for different pressures histories.

(a) SS1 (case 2, Table 1.3).

(b) Slug flow (case 14, Table 1.3).
Discretization

- Moving grid method between nodes 1 and $N-1$.
- Variable time step (method of characteristics), satisfying CFL (Courant–Friedrichs–Lewy) condition.
- Implicit scheme, predictor-corrector method for non-linearities.
Simulations

Figure 1.20: Simulated limit cycle, pressure at the bottom of the riser (type SS1, case 2, Table 1.3).\[ T_{sim} = 80.4\, s \quad T_{exp} = 85.6\, s \]
Simulations

(a) 

(b)
Simulations

(c)

(d)
Simulations

(e)

(f)
Simulations

(g) x (m) vs. t (s)

(h) s_u (m) vs. t (s)
Figure 1.23: Experimental data (Table 1.3) and stability map for $P_s = 2.013 \text{ bara}$ and $T = 293 \text{ K}$. 
Figure 1.24: Experimental data (Table 1.4) and stability map for $P_s = 2.70 \, \text{bara}$ and $T = 293 \, \text{K}$.
Figure 1.25: Experimental data (Table 1.5) and stability map for $P_s = 4.10\, bara$ and $T = 313\, K$. 
Severe slugging flow regime maps

Figure 1.26: Stability and flow regime map for $P_s = 2.013\, bara$ and $T = 293\, K$. 
SS flow regime map (Tin, visual)
Linear stability analysis

• Although empirical stability criteria (Taitel, 1986; Pots, 1987; Jansen, 1996) are useful for a first estimation of the unstable region (they are even used in commercial steady-state computer codes), a common drawback is that they were not derived from complete dynamic system models, but from ad-hoc conditions in which many physical effects were disregarded; consequently, their applicability is quite limited.

• Linear stability analysis is a more efficient alternative to time domain simulations to identify the stable and unstable regions.

• Other application areas in multiphase flows: flow pattern transitions, boiling crisis, density-wave instabilities.
Linear stability analysis

- Eliminating the void fraction from the algebraic relation, riser equations can be written as:

\[ A_m(v) + B_m(v) \cdot \left\{ \frac{\partial v}{\partial t} \right\} + C_m(v) \cdot \left\{ \frac{\partial v}{\partial s} \right\} = \{0\} \; ; \; v = \begin{pmatrix} j_g \\ j_l \\ P \end{pmatrix} \; ; \; 3 \times 3 \]

- Linearization \( v(s, t) = \tilde{v}(s) + \hat{v}(s, t) \), yields the continuum perturbation equations:

\[ A(\tilde{v}) \cdot \hat{v} + B(\tilde{v}) \cdot \left\{ \frac{\partial \hat{v}}{\partial t} \right\} + C(\tilde{v}) \cdot \left\{ \frac{\partial \hat{v}}{\partial s} \right\} = \{0\}; \; \hat{v} = \begin{pmatrix} \hat{j}_g \\ \hat{j}_l \\ \hat{P} \end{pmatrix} \; ; \; 3 \times 3 \]
• Integrating in position and adding the linearized boundary conditions (pipeline equations, choke valve), yields the discretized perturbation problem:

\[
\begin{align*}
\mathbf{G}^*(\mathbf{\tilde{v}}) \cdot \left\{ \frac{\partial \mathbf{v}}{\partial t} \right\} + \mathbf{H}^*(\mathbf{\tilde{v}}) \cdot \mathbf{\tilde{v}} &= \{0\}; \\
\mathbf{\hat{v}} &= \begin{bmatrix} jg \\ j_l \\ \hat{P} \end{bmatrix}; \quad 3N \times 3N
\end{align*}
\]

• System stability depends on the characteristic polynomial:

\[
\det(\lambda \mathbf{G}^* + \mathbf{H}^*) = 0
\]

• \( \mathbf{G}^* \) is singular in NPW model, resulting \( 2N - 2 \) finite eigenvalues.
Figure 4: Eigenvalues well-posed behavior in the vicinity of the stability condition, parameters from Taitel et al. (1990).
Figure 5: Stability map for a vertical riser with different nodalization, parameters from Taitel et al. (1990).
Figure 6: Stability map for a vertical riser with different buffer lengths, parameters from Taitel et al. (1990).
Parametric study $j_{g0,l}$

Figure 7: Stability map for a vertical riser with gas injection at the bottom of the riser, parameters from Jansen et al. (1996).
Figure 8: Stability map for a vertical riser with choking at the top of the riser, parameters from Jansen et al. (1996).
Figure 9: Stability map for a catenary riser for different separation pressures, parameters from Wordsworth et al. (1998).
Publications


Papers at JEM 2015

Azevedo, G. R., Baliño, J. L., Burr, K. P., Linear stability analysis for severe slugging including inertial effects.

Yamaguchi, A. J., Baliño, J. L., Experimental study of severe slugging in a air-water pipeline-riser system.

Burr, K. P., Baliño, J. L., Evolution equation for weak hydrodynamic instabilities of two-phase flows in pipeline-riser systems.
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Thank you!